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CLARENCE AUGUSTUS CHANT

By John F. Heard

The Royal Astronomical Society of Canada mourns the loss of its Editor and the oldest link with its origins. Dr. C. A. Chant, a member since December 13, 1892, President from 1904 to 1907, Editor since 1907 of the Journal and of the *Observer's Handbook*, died at his home, Observatory House, Richmond Hill, on November 18, 1956 at the age of 91.

Dr. Chant was born at Hagerman's Corners, north-east of Toronto, on May 31, 1865. After attending Unionville Public School, Markham and St. Catharines High Schools and York County Model School, he taught grade school for a year before entering the University of Toronto. After graduating in the Mathematics and Physics course in 1890, he was appointed in the following year to the teaching staff in the Department of Physics of the University. Except for a year's leave of absence for graduate studies for his Doctor's degree at Harvard and several shorter leaves for expeditions and research, Dr. Chant taught continuously at the University of Toronto until his retirement in 1935. This was a momentous period in the history of Canadian astronomy; to Dr. Chant's foresight and initiative must go almost the full credit for the roles played in this development by the University of Toronto and the Royal Astronomical Society of Canada.

The Astronomical and Physical Society of Toronto had been incorporated in 1890. In 1892, though he had then little knowledge of astronomy, Dr. Chant attended a few meetings of the Society and became a member at the end of the year. Probably he was first attracted by the "physical" side of the Society, but it was not long before the study of the stars began to have a fascination for him which he could not resist. In 1903, as the Society was changing its name to the Royal Astronomical Society of Canada, Dr. Chant was pressing for improvement of the posi-

tion of astronomy in the University. In 1904, as a result of this urging, a sub-department of Astrophysics was established under Dr. Chant's direction, and a graduating course in astronomy was added to the curriculum. From this time onward, Dr. Chant's single purpose in life seems to have been to promote the study of astronomy.

In 1904 Dr. Chant was elected President of the Society, and, although the usual term of office was, as now, two years, he remained President for four years. It was during this time, and at Dr. Chant's urging, that the Society began to broaden out, altering its Constitution to enable it to embrace Centres in other parts of Canada and to accept members from outside the Dominion. During this same period Dr. Chant launched the new series of publications known as the JOURNAL and also the annual *Handbook*, of both of which he remained Editor for exactly fifty years.

In 1912 Dr. Chant, dissatisfied that the Society and the University lacked a large telescope, began a persistent campaign to acquire, for one or the other or for both, a major research observatory. He wrote articles,

Dr. Chant in the library of the David Dunlap Observatory.

gave lectures, submitted reports, besought authorities. A less determined advocate would have been discouraged by the many disappointments which he suffered over the years. However, in 1927, Mrs. Jessie Donalda Dunlap, impressed with Dr. Chant's enthusiasm, offered to present to the University an Observatory in memory of her late husband, David Alexander Dunlap, who had joined the Society in 1921 and who had expressed eager interest in the observatory project. In this way Dr. Chant's one-man campaign was rewarded, and on his seventieth birthday, May 31, 1935, the David Dunlap Observatory was opened officially and Dr. Chant, its first Director, retired, not to a life of idleness, but to continued activity, revising his text-books, editing the Society's publications and writing his memoirs.

Over the years, Dr. Chant's dedication to teaching exerted a major influence, not only on Canadian astronomy, but also in Secondary School education. His sub-department of Astrophysics became in time a full-fledged Department of Astronomy, and under him were trained most of the Canadian astronomers who were to develop Canada's present position in international astronomy—among them five directors of observatories. For the schools and colleges he was co-author of several well-remembered text-books: "High School Physics" and "Mechanics for the Upper School" with F. W. Merchant, and "A Text Book of College Physics" with E. F. Burton. His "Our Wonderful Universe" has appeared in three English editions and has been translated into five foreign languages, remaining still a classic of clear and simple exposition of basic astronomy. Part of his memoirs was published in 1954 by the University of Toronto Press as "Astronomy in the University of Toronto".

In research Dr. Chant's interests ranged from early experiments with Hertzian waves to the Einstein Effect at total solar eclipses. His "eclipse chasing" took him in 1922 as far as Western Australia where he and Dr. R. K. Young, then of the Dominion Astrophysical Observatory and later to be his colleague, obtained an early verification of the Einstein theory of gravitation through the deflection of starlight by the mass of the sun.

Dr. Chant was justly rewarded for his tireless efforts. The University of Toronto conferred on him the Honorary Degree of LL.D. in 1935. Harvard awarded him the Silver Medal of the Tercentary of 1936. He was Fellow of the Royal Astronomical Society of London, Fellow of the Royal Society of Canada and Fellow of the American Physical Society, and he had been Vice-president of the American Astronomical Society from 1934 till 1936. In his honour the Royal Astronomical Society of Canada established the Chant Medal award in 1940.

Until five years ago Dr. Chant was able to attend meetings of the Society, and the older members will recall his scholarly public lectures, his charm of manner and his great kindliness. For his manifest contributions to Canadian astronomy all must be grateful; professional astronomers owe him a debt as a great teacher and as the founder of the Department of Astronomy and the David Dunlap Observatory of the University of Toronto; all astronomers, amateur and professional alike, owe to him the unity and prestige which is now enjoyed by the Royal Astronomical Society of Canada.

CANADIAN SCIENTISTS REPORT

III. TIME CHANGES OLD AND NEW

By W. S. McClenahan

The expression "How Times Change" is not new. Officials employed in a national time service may think this should be altered to read "How Time Changes".

Changes in recording the passage of time can be noted in the development of the calendar. The Egyptian calendar is an example of a method of recording the passing years in which each year consisted of 12 months of 30 days, with 5 added days. Even then it was noted from the heliacal rising of Sirius that the natural year was about 365.25 days in length and the Egyptians realized that their three seasons, designated, Flood Time, Seed Time, Harvest Time, would gradually drift in the calendar year, requiring 1,461 calendar years before the seasons would return to the same part of the calendar year again.

Up to the time of the calendar reform by Julius Caesar, this appears to have been the only civil calendar in which the length of each month and each year was fixed by rule. Previously these were fixed by direct observation or left to a decision by an individual.

Other calendars such as those used by the Babylonians and the Greeks, had the beginnings of the months fixed by observation of the lunar crescent and consisted of twelve months. In order to keep the months closely related to the seasons of the year, one month was repeated when required.

Our present-day calendar is the result of changes made in the Roman calendar. The insertion of days and months had been neglected to such an extent that Caesar caused sixty-seven days to be added in the year 46 B.C. to get things straightened out again. The length of the year being approximately 365.25 days was allowed for by an edict to have an extra day inserted every fourth year. Apparently time problems were as confusing in those days as they are at present and through a misunderstanding the pontifices put in an extra day every three years. By 8 B.C. the calendar was again off the track by three days. This was painlessly corrected by abolishing leap years till A.D. 8.

Pope Gregory XIII introduced the next change in the Julian calendar by a bull published in 1582. The reason for the change at this time was to restore the vernal equinox to the position given it in the Easter Tables

formulated at the General Council of Nice in 325, namely March 21. The necessity for the revision came from assuming the length of the year to be 365.25 days whereas the correct value is 365.2422. By 1582 the vernal equinox was on March 11 instead of March 21. The Julian year was $0^{d}.0078$ or about 11 minutes too long, and from 325 to 1582 it is easy to calculate that this difference accumulated to the tidy sum of ten days. The adjustment was made on October 4, 1582, by calling the following day October 15 instead of October 5. To guard against a similar difficulty in the future only century years divisible by 400 were to be treated as leap years. Without this proviso by 1950 the vernal equinox would be out of step by approximately 3 days. This is allowed for very closely since 1700, 1800 and 1900 are not divisible by 400 and were not leap years. No changes have been made to this calendar since 1582.

History records the difficulties and delays in having the Gregorian calendar adopted throughout the world. It was not adopted in England until September 1752, and even then aroused bitter opposition and riots in which several persons were killed at Bristol. In recent years Calendar Reform is again a live topic and another change is not impossible.

The above remarks have to do with the recording of the passage of time. What about the unit of time? As a time standard either the mean second, which is 1/86400 part of a mean solar day, which in turn is the average length of all the days in a year, or the sidereal second which is 1/86400 part of a sidereal day, is being used. In either case if the rotation of the earth on its axis has changed, then the unit of time has changed.

Investigations have revealed changes in the longitudes of the sun, Mercury and Venus, that are similar to fluctuations in the longitude of the moon. A variation in the unit of time is blamed for these similar discrepancies between computed and observed positions and two causes may contribute to this variation: (1) A gradual slowing down of the rotation of the earth results from tidal friction in narrow seas. (2) If masses of material are displaced in the earth the moment of inertia will be changed and consequently the earth might speed up or slow down according as the moment of inertia is lessened or increased by such action.

In 1939 Sir Harold Spencer Jones investigated the above differences and concluded that these variations in longitude for the sun, Mercury and Venus were proportional to their mean motions and are similar to fluctuations in the longitude of the moon.

The above two possible causes of the erratic rotation of the earth have different effects on the sun and planets, than on the moon. Suppose a mass of material in the earth has moved farther out from the axis of rotation. This will cause the earth to slow down, consequently the sun.

moon and planets will all appear to be ahead of their computed places by the same amount of time.

Tidal friction which tends to slow down the earth, causes also a change in the orbital motion of the moon. Since the tidal wave results from the attraction of the moon, it in turn attracts and tends to accelerate the moon's motion. This causes the moon to recede from the earth and changes its period of revolution. Tidal friction has no direct effect on the sun or the planets.

The discrepancies between the observed and computed positions of the sun, Mercury and Venus have indicated that the length of the day has been slowly increasing at the rate of one millisecond ($0^s.001$) per century, the cause being the effect of tidal friction. From the performance of the more accurate quartz-crystal clocks, it is now well established that there is also a short-term period in the variation of the earth's rotation.

Some years ago when the Shortt Free Pendulum clock first came into use, the greater uniformity of its rate over the previous pendulum clocks resulted in a change being made in the determination of clock corrections. When such corrections were plotted, the effect of nutation on the first point of Aries was quite apparent. Naturally a clock with a uniform rate could not follow the irregular motion of this reference point from which all star right ascensions are measured. For this reason uniform sidereal time was adopted, in which the nutation in right ascension is removed from the observed clock corrections. This came into effect January 1, 1936, the correction amounting to approximately one second of time.

This change seemed to answer current needs for a revision of a time unit until the quartz crystal clocks came into use. Their very accurate performance indicated a variation that could only be explained by a rotation of the earth which was slower than average in the first part of the year and faster in the last half of the year. The earth stands condemned as a master clock which is unable to provide a uniform time standard.

At the eighth General Assembly of the International Astronomical Union held in Rome, September, 1952, the following resolution was adopted:

"It is recommended that, in all cases where the mean solar second is unsatisfactory as a unit of time by reason of its variability, the unit adopted should be the sidereal year at 1900.0; that the time reckoned in these units be designated "Ephemeris Time"; that the change of mean solar time to ephemeris time be accomplished by the following correction:

$$\Delta T = +24^s.349 + 72^s.318T + 29.950T^2 + 1.82144 \ldots B$$

where T is reckoned in Julian centuries from January 0, 1900, Greenwich mean noon, and B has the meaning given by Sir Harold Spencer Jones in

Monthly Notices R.A.S., vol. 99, p. 541, 1939; and that the above formula define also the second. No change is contemplated or recommended in the measure of Universal Time, nor in its definition."

The mean solar second is unsatisfactory for use in the national ephemerides since theory assumes time as invariable when computing the positions of the sun, moon and planets. A further I.A.U. resolution stated that Ephemeris Time would not be introduced until 1960.

It was later pointed out by Prof. A. Danjon of the Paris Observatory that the tropical year is a more fundamental unit. Its length is derived from Newcomb's tables of the sun while the length of the sidereal year is dependent on an adopted value of the constant of precession.

The Committee of Weights and Measures at its meeting in September, 1954, adopted the following definition of the fundamental and invariable unit of time: "The second is the fraction 1:31556925.975 of the tropical year for 1900.0."

The sun's mean longitude according to Newcomb's tables is:

$$279° 41' 48''.04 + 129602768''.13T + 1.089T^2$$

where T is in Julian centuries measured from the epoch 1900 January 0^{d}0.

If, then, the T is a changing quantity, a difference will show up between observed and computed values of the mean longitude. It was these differences in the case of the sun, moon and planets that were examined by Sir Harold Spencer Jones and led to the following correction to the sun's mean longitude:

$$1''.00 + 2''.97T + 1''.23T^2 + 0.074804 \ldots B$$

The reason for using B, a fluctuation in the moon's longitude, is because the moon has the fastest motion. For instance, the sun moves only one degree per day whereas the moon moves in its orbit about 13° per day. The variation in the rate of rotation of the earth is much easier to detect on the basis of discrepancies between observed position and computed position in the moon's longitude.

The correction ΔT comes from this equation. It can be found by a simple calculation since the sun moves 360° in 365.25 days it will change 1''.00 in 24^s.349. At first one might expect the ΔT to be zero for 1900.0, whereas the ΔT is:24^s.349 + 1.82144 B. B is given as approximately — 16'' for 1900 so the correction ΔT is approximately —4 seconds of time.

What will happen in 1960? Opinion is divided as to whether clocks should be adjusted to indicate Ephemeris Time. If this is done it will mean a correction of + 35 seconds approximately. Such a change could be absorbed into everyday life with very little notice. The fundamental ephemerides of the sun, moon and planets will all be tabulated with ephemeris time as argument and the *Improved Lunar Ephemeris* 1952–59 has already been issued.

PROCEEDINGS OF THE NATIONAL SCIENCE FOUNDATION CONFERENCE ON BINARY STARS

Preface

The National Science Foundation (United States) sponsored a Conference on Binary Stars at the Dominion Astrophysical Observatory, Victoria, B.C., on August 16, 17, and 18, 1956. This publication of the papers has been made possible through the co-operation of the Foundation and the Royal Astronomical Society of Canada. The arrangements for the Conference were expedited and made successful originally through the active support of Programme Directors Peter van de Kamp and Helen S. Hogg. The Steering Committee consisted of K. Aa. Strand of the Dearborn Observatory, Frank Bradshaw Wood of the Flower and Cook Observatory, and R. M. Petrie of the Dominion Astrophysical Observatory. The committee on local arrangements was Jean K. McDonald, Kenneth O. Wright, and Doris M. Sewell.

The National Science Foundation was represented at the Conference by Helen S. Hogg. The delegates to the Conference are listed below, with an asterisk distinguishing those who acted as Chairmen at the different sessions:

L. Binnendijk, Flower and Cook Observatory, Philadelphia

J. F. Heard, David Dunlap Observatory, Richmond Hill, Ontario

Su-Shu Huang, Department of Astronomy, University of California, Berkeley

G. E. Kron, Lick Observatory, Mount Hamilton

Albert P. Linnell, Amherst College, Amherst

*W. J. Luyten, Department of Astronomy, University of Minnesota, Minneapolis

Andrew McKellar, Dominion Astrophysical Observatory, Victoria

John E. Merrill, Princeton University Observatory, Princeton

G. J. Odgers, Dominion Astrophysical Observatory, Victoria

*Joseph A. Pearce, Dominion Astrophysical Observatory, Victoria

R. M. Petrie, Dominion Astrophysical Observatory, Victoria

L. Plaut, Mount Wilson and Palomar Observatories, Pasadena

D. M. Popper, Astronomy Department, University of California, Los Angeles

*Joel Stebbins, Lick Observatory, Mount Hamilton

K. Aa. Strand, Dearborn Observatory, Northwestern University, Evanston

*O. Struve, Department of Astronomy, University of California, Berkeley
*G. Van Biesbroeck, Yerkes Observatory, Williams Bay, Wisconsin
O. C. Wilson, Mount Wilson and Palomar Observatories, Pasadena
Frank Bradshaw Wood, Flower and Cook Observatory, Philadelphia
Charles E. Worley, Lick Observatory, Mount Hamilton
K. O. Wright, Dominion Astrophysical Observatory, Victoria

The preparation of these proceedings for publication was carried out by Jean K. McDonald.

R. M. PETRIE

REMARKS ON THE STUDY OF DOUBLE STARS
AN INTRODUCTION

By R. M. Petrie

The purpose of these remarks is to provide an introduction to the discussion of problems of double-star astronomy. An important branch of astronomy since the time of Herschel, the study of double stars is still active today and still makes valuable contributions to the subject. Its problems continue to challenge equally the observer, the computer, and the theoretician.

The primary aim of this conference is to explore ways of maintaining, and even increasing, activity in the study of double stars. This is to be done by discussing problems now being attacked, by considering programmes which should be initiated or expanded, and by noting those projects where some degree of co-operation among astrometrists, photometrists, and spectroscopists is desirable. A number of problems requiring attention by one or more branches will now be reviewed very briefly.

Stellar Masses and the Mass-Luminosity Relation

Direct information on this fundamental question comes from two sources: (a) the orbits and parallaxes of visual binaries, and (b) the combination of photometric, and spectroscopic, data of eclipsing pairs. The number of visual binaries for which orbital elements may be obtained in the reasonably near future is, of course, limited, and apart from the increase in precision now being effected, no large addition can be expected from this source. Spectroscopic observations should be made wherever they may help in the derivation of the orbital elements or the masses of visual binaries and it would probably be useful to obtain high-dispersion spectrograms of the components of wide pairs. On the other hand, one would like to see a more vigorous application of the double-star interferometer to spectroscopic binaries of long period. An inspection of the catalogue gives some ground for believing that a few "*Capellas*" may be found!

The richest source of information on the more luminous part of the mass-luminosity relation lies in the eclipsing binaries. Work is needed here because the number of well-known massive systems is small. Furthermore, some of the high-temperature massive binaries are afflicted

by anomalies in the light, or velocity, curves which detract from their usefulness as standards of mass and luminosity. Modern developments have placed many, perhaps hundreds, of eclipsing systems within reach of the photometer and many light-curves have been obtained. But the output of spectroscopic orbits for stars fainter than apparent magnitude 8.0 is small. Here, surely, is an opportunity to strengthen our knowledge of stellar luminosities and dimensions. Is it not time for spectroscopists using our largest telescopes to undertake observations of faint eclipsing binaries with spectrographs of moderate dispersion?

We cannot expect greatly to increase the number of visual binaries with well-determined orbital elements unless we wait for several decades but we may hope to make significant additions by employing the differential method as suggested many years ago by Lundmark and Luyten. In this method one measures the mass ratios and magnitude differences, which data give the slope of the mass-luminosity relation and the relation itself very readily. The method enjoys the great advantage of being relatively insensitive to parallax errors. It was possible a few years ago to apply this method to spectroscopic binaries with very satisfactory results, judged by the independent data from eclipsing systems.

There appears to be an opportunity here. Measures of Δm offer no difficulty to modern photometric techniques if the components of the visual binary are not too close together. Mass ratios may be found from meridian-circle observations of both components, from the photographic method as developed recently by Strand, or, in some cases, from spectroscopic observations of radial velocity, without waiting for the stars to describe a complete orbital circuit. We should not neglect this chance of improving our knowledge of the relation, or relations, between mass and luminosity. In passing it may be noted that differential measures on visual pairs are valuable also in studying spectrum-magnitude diagrams and evolutionary changes.

Wide Pairs

Visual binaries whose components are separated by, say, more than 5″ are usually of secondary interest to astrometrists but they are very useful objects nevertheless, because the components may be observed separately with the spectroscope. They may then serve as standards to check our systems of radial velocity and to calibrate spectroscopic criteria of absolute magnitude. In these applications we need pairs with components differing in spectral type but not differing very much in brightness. A fine example is A.D.S. 2559; 7.4, B3; 8.5, gG5; $\rho = 3''.8$.

Before using well-separated pairs as spectroscopic standards one must,

of course, verify that the systems are true binaries and not optical doubles. The *a priori* probability of physical connection is always overwhelmingly affirmative but certain observations cause uneasiness. For example, out of eighteen binaries recently examined for suitability as radial-velocity standards five systems gave radial-velocity differences, between the components, as follows:

A.D.S.		ρ	m, Type	ΔV km./sec.
1262		25″	7.1, B5; 8.2, G8	5.5±1.0
7654	α Leonis	176	1.3, B7; 7.6, K2	12.6±4.0
11910		19	6.1, B5; 8.4, A2	7.1±2.1
12197	η Lyrae	28	4.5, B5; 9.0, A1	9.3±1.4
13524	κ Cephei	7.5	4.4, B9.5; 8.4, A8	4.5±1.3

It may be remarked that the spectroscopic absolute magnitudes of the components are in accordance with the assumption that the pairs are physically connected. Have we evidence here of triple, or multiple stars, or are these systems merely optical pairs? Continued astrometric measures on these and other wide pairs will supply the answer and be of value in spectroscopic studies.

The anomalies discussed above are not confined to wide pairs but may appear also among the physical systems with known orbits, as indeed is the case with 61 Cygni. For example, radial velocities obtained at Victoria by Miss Underhill of ϵ Hydrae AB, α Ursae Majoris and ζ Herculis show scatter in the annual series five to ten times that to be expected from the dispersion and quality of the spectral lines. It is premature to advance an explanation, but perhaps we have in these cases further evidence of multiplicity.

Eclipsing binaries offer a bewildering variety of disagreements between the photometric and spectroscopic data. Indeed, one seldom finds the two sources agreeing in all respects in describing a system. The well-known examples of β Lyrae, RW Tauri and U Cephei are probably explainable in terms of circumstellar gases. Other cases are perhaps not disposed of so easily. An interesting group is represented by AO Cassiopeiae and V448 Cygni, where one or both components exhibit rather similar departures from elliptic motion. The stars are very hot, very massive, and very bright but the components are well separated from one another. The light curves are satisfactorily smooth and conventional.

Less spectacular, but equally puzzling, are stars like S Equulei where the radial velocities refuse to correlate with the smooth, Algol-type light

curve and AR Cassiopeiae where the spectroscopic apsidal motion disagrees with the photometric value.

One suspects that a large number of anomalies exist although some may be removed by better observations. Systems showing peculiarities and discordances should certainly be attacked simultaneously with photometer and spectroscope and more detailed spectroscopic studies should be made. The explanation of the anomalous results where they are definitely established will contribute to our understanding of the outer layers of stars generally.

Composite Spectra

Certain of the composite spectra originate in binary systems intermediate in separation between visual and spectroscopic pairs. These are spectra classified as I by Hynek and in which the composition is described as G + A, K + A, or sometimes K + B. The distinguishing features are (a) a progressive weakening of the solar-type lines from about H_γ toward the violet, and (b) a relatively narrow *K* line in a region of almost continuous background quite different from what one would expect from the appearance of the spectrum at, say, λ4300.

The selection effects are ferocious since composite spectra will be recognized only when the component stars fall within rather narrow limits of magnitude, and spectral-type differences. The field promises to be a rich one, however, as we may measure mass ratios and magnitude differences and, with patience, orbital elements may be derived. One recalls that ζ Aurigae, 31 Cygni, and VV Cephei were originally known merely as composite systems, and yet how much they have contributed to astronomy!

The number of composites for which orbits may eventually be known appears to be substantial. In a survey begun recently at Victoria, out of thirty systems examined, nine show a difference in the radial velocities derived from the solar, and the A-type, lines. One system (discovered earlier by McLaughlin) showed a periodic doubling of the narrow *K* line thus proving that the system is triple! In addition, seven stars gave evidence of velocity variation in the solar-type component. The above results, not all original discoveries in this survey, give unmistakable evidence of the existence of a fruitful field of research. Spectroscopic observations should come first to be followed by photometric and astrometric measures when indicated by the radial velocities. The following stars have shown differences of more than 30 km./sec. between the velocities derived from the solar-type lines, and from *K* and Hδ. I am indebted to D. H. Andrews for many of the radial-velocity measures.

H.D.	m_v	Type	K,Hδ−Solar	Plates
4775–6	5.45	F2, A2	49 km./sec.	1
9352–3	6.05	K0, A0	45	11
19926–7	5.84	G5, A5	57	1
157978–9	5.98	A0, G	triple star	many
177390–91	7.9	A0, G	33	1
184759–60	5.42	F5, A0	38	1
186518	6.56	K0, A0	34	1
196093–4	4.85	K5, A3	49	1
196753–4	6.13	K0, A3	31	1

Composite systems of the sort considered here offer an opportunity of detecting under-luminous stars because these systems are composed of a solar-type star and one of earlier type, approximately equal in brightness at about λ4000. If the solar-type component were to be a main-sequence star then the early-type component must be several magnitudes fainter than normal for its spectral type. The main problem is to estimate the spectral type and luminosity of the solar-type component, a task made difficult by the superposition of two spectra. A survey made by the writer some years ago suggested the existence of a number of sub-luminous A stars but the results were not conclusive because of the above-mentioned complications.

Persistent spectroscopic observations of composite systems can hardly fail to advance our knowledge of double stars and to make astrophysical contributions. One suspects that a number of ζ Aurigae stars are even now awaiting discovery.

Other Topics

Space does not permit of an exhaustive compilation but one or two matters may be mentioned in closing. It is not necessary to discuss the ζ Aurigae stars except to point out that their importance to astronomy can scarcely be over-emphasized. Accurate photometry of these systems, in several colours, is the prime need. It will probably be worth while to arrange for world-wide continuous coverage, photometric and spectroscopic, during the critical periods before and after totality at future eclipses.

Some important topics do not appear on the programme. The role of photographic photometry is not included specifically nor is the interferometric measurement of very close pairs, nor the application of photoelectric photometers to the measurement of Δm in visual binaries. These matters should not be overlooked in planning work in the field of double stars.

Theoretical aspects of our subject are being represented at one session.

It appears from current studies that we may expect rapid and stimulating progress in attempts to explain the origin of binary stars and the evolutionary processes. This conference makes it possible to bring together those engaged in observational studies and those pursuing theoretical work. Out of their mutual challenges, and recriminations, new programmes and fruitful contributions will evolve. Progress will be steady and firm if a close liaison is maintained among the different branches of our subject.

PHOTOELECTRIC OBSERVATIONS OF VARIABLE STARS

By Gerald E. Kron and Katherine C. Gordon

The art, or science, of following changes in the light from variable stars has been practised for nearly 300 years. Over a century ago, methods were devised to yield measurements having fairly small experimental errors, but freedom from systematic errors could never be guaranteed. When electrical methods were introduced late in the nineteenth century (1895), the way was opened for the final significant improvement, made in the early part of this century: the introduction of the emission-type photocell. This type of light-sensitive element has been found to be an ideal detector, as it combines absolute linearity of response with the highest sensitivity of any known detector of light. Modern electrical photometry has resulted simply from refinements in equipment and methods.

The development in this country of the use of the emission cell dates from the measurement of the light of the solar corona during the eclipse of June 8, 1918, by Jacob Kunz and Joel Stebbins (1919). Later, Stebbins used photocells produced by Kunz for photoelectric observations of eclipsing binaries. In so doing, he established methods and standards still useful and impressive today. Even now, traditional techniques, when applied to the photometry of intrinsic variables, yield observations of such high precision that nothing better is desired. It is to be understood, therefore, that the refinements discussed here are applicable mainly to the observation of eclipsing binaries.

The precision of photometric observing is related to the signal-to-noise ratio of the measurements, the efficiency with which the observer uses his available time, and the effectiveness with which the observer can correct for such unwanted phenomena as background light and atmospheric extinction. Each of these error categories is complex, in that each contains more than one contributing factor, usually unrelated to each other. In the following discussion, these sources of error will be considered in some detail, and emphasis will be placed on points that have not been discussed in the past.

The noise level of photometric meter deflections is made up of random components arising from the shot noise of the photocurrent, the variations in light intensity caused by scintillation, and the variations in light caused by random fluctuation in atmospheric transparency. It has been shown (Kron, 1940) that with a 36-inch telescope, used for photometry with a

Kunz potassium-hydride photocell, the shot and scintillation (seeing) noise were equal for a star of 8th magnitude. Since those days, more efficient photocells have come into use, and this equality is reached at about 11th magnitude. Hence, for eclipsing variable stars most commonly observed, the shot noise component will be negligible; when it is not, the shot noise contributes a random component that will reduce precision in a way that has been discussed thoroughly in the literature (Smith, 1932; Johnson, 1948). The effects of scintillation can be reduced by observing at small air masses when possible, and at sites known for their good seeing. An apparent improvement can be obtained by employing a long time-constant, but this is illusory, as the efficiency actually suffers. Though the dark or sky deflection does not suffer from scintillation, the observer using a long time-constant is bound to spend as much time observing these relatively noise-free deflections as he spends observing the star. Noise from random variations in transparency is not well understood. If noise from this source has frequency components comparable with those in scintillation, then the same considerations apply. If, on the other hand, transparency variations are relatively slow, then their effect can be reduced by reasonable increases in the frequency with which the comparison and variable stars are compared. Differential methods such as those described by J. S. Hall (1953) should be useful for reducing the effects of atmospheric sources of noise. The degree with which differential methods will help depend upon the similarity with which the comparison and variable stars are influenced by the unwanted effects, the degree of perfection with which the differential method employed achieves simultaneity in the comparison, and upon any new unwanted effects that might be introduced by the differential method. The use of true differential methods for observing variable stars is still practically untried, and no encouraging performance data are yet available.

The efficiency of photoelectric observations made by conventional methods is much affected by the selection of the comparison star. Considerable time is lost if the comparison star is so distant that the telescope must be unclamped and moved from one star to the other. There is a great advantage in selecting a comparison star close to the variable, say within 5 to 20 minutes of arc, for transfer can then be made with the slow motion controls in a length of time equivalent to the exposure time, and only one exposure will be lost during the transfer. Selection of suitably close comparison stars is usually not possible when the variable star is bright (a noteworthy exception is YZ Cassiopeiae). Hence, if possible, the observer should try to choose variables in the 8 to 10 magnitude range,

bright enough so that the photometry will be seeing limited for moderate-sized telescopes, faint enough so that an abundant supply of satisfactory comparison stars will usually be available.

Observing efficiency can be increased by special optical devices, such as the prism device employed at the Lick Observatory for bringing the light from comparison and variable stars into the optic axis of the photometer simply by flipping a prism through 90° (Kron, 1939). By means of this apparatus, the dark or sky level need not be measured any more frequently than necessary. It was shown experimentally that in six minutes of observing time, an observation could be obtained of weight equal to one requiring 16 minutes of observing time without the device. The lack of absolute similarity between the two light paths may introduce small systematic errors, so the observer has to decide between the two evils: systematic error on the one hand *versus* increase in random error caused by loss in efficiency on the other.

Increased observing efficiency can be obtained by making the exposures and the transfers from star to star in a definite rhythm set by an automatic timer not under the control of the observers. In our experience, time intervals of either ten or fifteen seconds are both convenient and efficient. If one makes long exposures, errors mount because of the demands made upon the guiding, and because of the greater vulnerability of the observations to variations in atmospheric transparency. Very short exposures reduce the efficiency, owing to the more frequent transfers and references to the zero.

Important considerations in precision photometry are the treatment of differential atmospheric extinction, and the correction for background light from the dark or moonlight sky. The differential extinction is corrected by applying appropriate differential formulae with experimentally determined extinction factors. If the comparison star is close to the variable, this correction will be small, and even rather large errors in the extinction coefficient will not seriously affect the accuracy of the results. For example, at the large zenith distance of 60° (equivalent to two air masses) the differential extinction in blue light between two stars that differ in zenith distance by 15′ will be only $0^{m}.005$. Here is, therefore, a second reason for selecting a near-by comparison star.

The effect of background light may be reduced by employing a smaller focal plane aperture, but as this aperture is decreased in size, errors caused by imperfect guiding become more important. If an aperture with a diameter of 30″ is used, the yellow deflection on a star of tenth magnitude will be 97% starlight and 3% skylight for a natural dark moon-

less sky. If errors no more than 0.1% are to be permitted, then corrections for the background light must be made with an accuracy of 3% unless the comparison and variable stars are nearly equal in brightness. The traditional way to correct for background light is to measure the sky brightness every half hour or so, and then correct the deflections on the basis of an interpolation for the sky brightness at any given time. This method can lead to unnecessary error, especially if the observations are made during moonlight. Attempts to improve precision by making more frequent separate measurements of the sky decrease the efficiency. A better way to correct for background light is to refer to the sky level instead of to dark for each deflection, by moving the entire photometer to a sky sample just next to the star. A photometer incorporating a slide and adjustable stop mechanism for permitting a motion of this type was built at the Lick Observatory in 1947. The method has been found to be highly successful, especially for the observation of fairly faint stars during moonlight. Greater demands are made upon the guiding, as reference to the dark for guiding purposes is less frequent; experience indicates, however, that the precision gained from the method of sky correction is more than that lost by poorer guiding. In addition, the reduction of observations is simplified because the sky correction is included in the deflection.

Photometric errors caused by less than perfect guiding are in a class by themselves, and have not been discussed in the literature. Errors of this type have two sources, from inconsistent centering on optical parts of non-uniform transmission, and from variable rejection of light in the focal plane aperture. Optical parts not near the focus, such as the Newtonian flat, are generally harmless, because of the large size of the optical part in comparison to the guiding error motion. Two parts, however, the filter and the field lens, are usually close to the focal plane. It is well to keep to a minimum the number of optical parts close to the focus. Our practice is to have only a filter and a field lens; a window to the light receiver is eliminated by using the field lens in its place. The filter could be placed at some considerable distance from the focus, but when this is done the filters are large and clumsy, especially if the telescope is of small aperture ratio. The envelope of the multiplier can be at a good distance from the focal plane if the field lens has a sufficiently long focus, but this places a restriction on the minimum tolerable cathode size. We use cathodes no less than 1 cm. in diameter when possible, with about a 5 cm. focal length field lens in the f/5 beam of an ordinary reflecting telescope. The 1P21 has a cathode so small that we feel it can

not be used at its best performance with an f/5 telescope. Larger focal ratios simplify the problem and are to be preferred, provided that modification of the focal ratio does not introduce a new problem just as bad. We prefer a simple plano-convex fused quartz field lens, free from flaws. Filters are selected to be free from flaws too; those that are opaque to visible light are examined by photometric methods.

In order to learn more about guiding errors caused by light lost in the focal plane aperture, we have made measurements of the amount of light lost in apertures of different sizes. The measurements were made on a summer night of average seeing at Mount Hamilton, with the red, blue, and violet filters of the 6-colour photometer, with an infra-red sensitive Lallemand multiplier. The photometer was focused by using the focal plane aperture for a knife edge in a Foucault test for each filter, and for each aperture. The diameter of the stellar image looked to be about 1″.5 when viewed through the 4″ aperture. The light lost was found to be independent of colour, and is given in the following table. Obviously, errors caused by imperfect guiding will become larger as the focal plane aperture is decreased in size; on the other hand, errors from background light increase as the aperture is increased. We were able to observe the 9th magnitude eclipsing variable UX Herculis during full moon through a 20″ aperture with the satisfactory accuracy of $\pm 0^{m}.005$ a.d. per three-minute observation. For less sky illumination or a brighter star, the aperture could be increased. We found that guiding through the 20″ aperture under our conditions of observing was critical, and that the

Light Lost through Focal Plane Apertures

Diameter Aperture	Light Lost
45″	0.0%
30	0.5
20	1.5
14	2.2
10	3.2
6.7	6.3
4	11

precision was affected by the care used in centering the star just prior to the series of three exposures.

We feel that guiding is now our chief difficulty, and the next step in improving our equipment will be to provide for continuous guiding. This will be done by deflecting about 20% of the light from a point well ahead of the focal plane into a guiding eyepiece. The observer at the telescope will then be fully occupied, and his already exacting work will

then be harder than ever. A second observer will be a necessity, but we have always favoured having two observers, anyway. We are convinced that the personal comfort associated with not overworking the personnel contributes to accuracy, and that the observers in critical photometric work should follow through from gathering the observations to their final reduction and discussion.

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AVENUES OF PROGRESS IN INTERPRETATION OF LIGHT CURVES

By John E. Merrill

The importance of the contributions which studies of eclipsing binaries can make to our knowledge of absolute dimensions, masses, surface brightnesses, and mean densities of stars, has long been recognized. In more recent times their possible contributions to furtherance of our understanding of stellar internal structure, of photospheric and atmospheric phenomena, of "thick atmospheres" or "edgelessness", of evolution, have been added to the list. We find ourselves now at a point in the developing history of the field where it is peculiarly appropriate to assess both the benefits we can hope to derive from more concerted study of these stars and our facilities for most effective work.

Our basic facilities for solution of light and velocity curves of conventional "edged" stars are in good shape; both theory and corresponding tabular and graphical material are, generally speaking, somewhat beyond our immediate needs for handling presently available observational curves on the much-idealized Russell and Kopal models. How to use these means properly, to produce satisfactory, realistic interpretations of actual observed curves, looms as of more and more importance as system after system, under careful and precise observation with modern photocells and spectrographs, shows departures from the curves for idealized models (some probably transient, some almost certainly not) which have aptly been called "perturbations" and "complications" in the case of light curves. It appears that two additional sets of tables (neither at all extensive) should be provided in the fairly near future to aid in interpretation of light curves:

(1) Tables of the correction to α regarded as a function of ψ (or vice versa) resulting from the assumption of limb darkening varying with $\cos^2\gamma$, for use near the tangencies;

(2) A considerably streamlined table of the Kopal associated α-functions, when it becomes reasonably clear which probably enter significantly into actual light curves.

I have been planning, in a tentative way, to provide the first-mentioned tables after the Princeton Contribution on Illustrative Examples is finished.

Tabular material for discussion of light curves of stars in the "thick-

atmosphere" or "edgeless" category is needed now and will probably be needed more and more in the relatively near future; in the early stages of investigation of this fascinating phenomenon, it should be rather restricted in extent, because the most profitable choice of parameters for solutions (an intriguingly complex problem here) is still far from defined in either its observational or its theoretical aspects.

The Princeton nomographs are showing themselves capable of determining (as a mathematical exercise) the darkening parameter with a precision beyond that physically meaningful in most systems at present, in view of uncertainties as to the proper form for the law of reflection, the relative significance of the various theoretical distortional perturbations, and the treatment of the "complications" which turn up in thoroughly and precisely observed light curves.

In the light then, of the limits and deficiencies of interpretation evident on the solutional side, I should like to discuss briefly some areas in the study of eclipsing binaries in which closer integration of effort would be of special value in reducing these deficiencies, grouping my remarks under the rather rough headings of:

(1) emphases in programme selection,
(2) standards,
(3) co-operative programmes.

(1) *Emphases in programme selection.* We must admit, in all honesty, that we do not at present know on what basic astrophysical parameter or parameters the coefficient of limb darkening is most directly dependent, and also that the proper form of the law governing the reflection effect is uncertain. It is well known that the effects of limb darkening and of ellipticity of figure are badly entangled in a light curve, especially (and most unfortunately) near internal tangency, otherwise a very favourable region for darkening determination; it is perhaps not so well recognized how badly a significant degree of ellipticity may interfere with reliable determination of reflection effects, or how seriously the presence of significant "sine terms" must weaken our confidence in the physical validity (in the individual case) of the conventional interpretation of the coefficients of their corresponding cosine terms. Progress in the disentanglement of these factors (ellipticity, reflection, limb darkening) can probably be achieved fastest by increased emphasis on observation of systems selected (as well as may be in advance) for their comparative "simplicity", that is, for probable fairly close conformity to the Russell Model with a small ellipticity. There are some thirty-odd systems in Wood's Finding List (including some too faint for most equipment and

half-a-dozen requiring co-operative effort) for which the light curves should (barring complications) be affected mainly by limb darkening only and should therefore help to clarify its dependence relations.

Now that linearity of response of photoelectric equipment is well established, observation of more systems with little ellipticity but appreciable reflection (i.e. "Algol-type" systems) in three colours, would be very helpful. Since the differential reflection presumably varies considerably with the wave-length of observation in such systems one of the curves should be done in the red. Simultaneity of observation is easily attainable for curves in the UBV system if the U curve can be obtained with the given telescope at all; the red curve should be observed contemporaneously at least.

It is probable that for the immediate future the curve beyond the Balmer limit is less important than the curve in the red, but it is also probable that in the long run U, B and R or U, V and R would yield the greatest return.

This need of curves in the red is so pressing that the ideal requirement of contemporaneity of observation should be set aside (after all, we have not hitherto held to it!) in one group of cases in particular, and red curves obtained as soon as reasonably possible; this group comprises the bulk of the systems in Kopal's I.A.U. list of reliable photoelectric curves, with two or three of Dugan's stars added thereto, and one or two others for which blue series have been completed since Kopal's list was made up. A three-dimensional model of deduced limb darkening as a function of known spectral type and effective wave-length of observation strikingly exhibits both the serious deficiency of spread of the present material in the independent parameters and the vexatious scatter of the values so far deduced for the one assumed to be dependent on them. It is clear, in particular, that red curves are vitally needed, and will be especially valuable, for the blue stars.

This same model stresses our need of red dwarf systems; even one or two systems with one or both of the components of late K or early M dwarf type, would add to our knowledge far out of their numerical proportion in the group of well-observed curves. It is quite probable that the proper-motion surveys turn up no K or M dwarf prospects for us (Luyten stated at the Conference that it is not merely probable, but certain) and on the basis of the information available on periods and spectra only two systems in the Finding List (besides YY Geminorum) seem indicated as possibilities. The serious deficiency in spectral information (of which I shall speak later) leaves, however, the clear possibility that a search

programme based on obtaining spectra of the many Finding List systems now lacking them, might net us the one or two red dwarfs so badly needed.

My last point on emphases in programme selection has to do with coverage of the curve once the star has been put on the programme. It is absolutely necessary, for real precision of interpretation, that the *entire* curve be covered, in general covered twice, in each colour, though by thoughtful integrating and real simultaneity of observing in two colours or in three, the total number of runs can sometimes be reduced somewhat. It is also very important that the entire observational series on each star be compressed into the shortest time-interval reasonably possible, whenever practicable into one observing season at most. In the days of the visual photometer the observer felt that he needed variety in body positioning and muscular strains to keep his observing eye in proper condition for making valid judgments of equality of brightness of images, so he deliberately carried several stars at once on his observing programme to get this variety. Operation of a photoelectric photometer does not impose this particular demand on the observer and its greater inherent precision has caused the "complications" in curves to assume a larger role in our thinking.

The obvious difficulty is that we are not at present in a position to know to which class, transient or permanent, to assign a complication that shows itself in a given curve. We need, then, two (or three) successive "epochal light curves" of the one system, closed series covering separate short time intervals, rather than one curve for each of several systems, smeared out over the same total calendar interval, to have the best prospect of realistic interpretation.

(2) *Standards.* It is immediately clear, however, that efficiency in production of "epochal light curves" having overall reliability well in line with the precision of measurement possible with modern equipment, entails two further improvements in our general practice. First, the time has now come for adoption by our photoelectric observers of a field, or set of fields, of stars, as standards of magnitude and colour. Observers of eclipsing binaries should calibrate their photoelectric equipment against these standards, publishing results; telescope-plus-filter combinations should be brought as closely as is reasonable onto the UBV system and the residual conversion factors to UBV stated. Johnson's determination of magnitudes and colours for stars in Praesepe satisfies a basic need in our field; we should make use of his work, and thankfully.

Second, the task of providing adequate comparison stars in eclipsing binary fields should hereafter be assumed mainly by others than the

light-curve observers themselves. With Johnson's standard fields and with good filters available the dangers which formerly faced us, stemming mainly from differences in colour sensitivity and in sky, are almost entirely eliminated. Comparatively modest photoelectric equipment on (say) college telescopes devoted mainly to teaching, can do and should henceforth be doing most of the work of establishing adequate and reliable comparison stars. The initial list of stars in a given field to be investigated for constancy should still be made, naturally, by recourse to the plate collections, but then a considerable number of photoelectric runs of two or three hours each, interlinking those initially listed, should be carried through *and published.* This work is not as glamorous as construction of light curves themselves, but would be a very real contribution, of permanent value, because it would be standardized. It might be noted that the observations on a given comparison-star sequence should definitely not be limited to a single observing season, and can be made very effectively at stations where good sky conditions do occur, but too erratically to make periodic variables themselves a wise choice of programme.

When practicable, three adequately validated comparison and check stars in each field should be the objective, not all necessarily as nearly of the magnitude of the variable as used to be required. Two of these should be used in observation of the variable, one of the two serving at the same time as check star on the constancy of the other, principal, standard; the third validated comparison star is then a reserve, making it possible to continue the observational series on the light curve without serious interruption if, unpredictably, either of the first two shows itself unreliable.

(3) *Co-operative programmes.* Standardization of equipment should make practicable the direct combining of photometric data obtained by different observers for the same star, with greater safety than heretofore. Acting for Commission 42 of the I.A.U., Wood has interested a number of observers in a world-wide effort to obtain this year eclipse curves and spectrograms for the very long-period systems ϵ Aurigae and VV Cephei. The response was particularly gratifying in that several who had not intended to work on either star joined in the programme and thereby closed troublesome longitude gaps in the distribution. With the need of reliable data on thick-atmosphere stars so apparent, like programmes should be set up for other long-period systems.

Similar longitude-wise arrangements should be made for observations of eclipsing systems with periods very close to one day or to multiples of one day. There are over forty stars in the Finding List for which such

observing could produce epochal light curves of great potential value, but for which the overall time-interval required for proper curve coverage at one station is far beyond reasonable limits.

But perhaps our most pressing need for more and closer co-operation appears when we consider the paucity of spectroscopic information at hand for eclipsing binaries. We in the field, not the spectroscopists, must be held responsible for the fact that for over 400 of the Finding List systems, *no spectroscopic information whatever is presently available.* It is true that this deficiency is confined to objects of magnitude 9.0 and fainter, but it is not by any means entirely at the faint end of the range; perhaps the most surprising thing is that the situation is not significantly different numerically in the two hemispheres as far as the general data needed are concerned. Since half of the eclipsing binaries brighter than magnitude 9.0 are already listed as showing two spectra, enormous return could be expected from increased effort to provide spectroscopic material.

For better planning of photometric programmes spectral classes are vital. Identification of Finding List stars on existing objective-prism plates (e.g., Vyssotsky's) would be a routine matter; an expert who would make approximate classifications for the stars identified would perform a service of great value to the whole field of eclipsing binary research. For determination of absolute dimensions more radial-velocity curves of selected systems are needed. In the future we should make special effort to see that the radial-velocity data for selected systems are obtained at least contemporaneously with the photometric, in view of the peculiarities and complexities which are making their appearance on plates taken with high dispersion. When strict simultaneity of spectroscopic and photometric observation becomes practicable, with the advent of the National Photometric Observatory, we can reasonably expect much more rapid progress in interpretation of these surficial phenomena.

Where then lie our most promising avenues for making the study of eclipsing binaries contribute most effectively to the advance of knowledge? In closer and wider co-operation among theoretical, spectroscopic and photometric workers, for planning and executing balanced programmes; in facilities for carrying through such programmes, enlarged not only by the utilization of existent equipment not presently in fullest use but also by the construction of new equipment imaginatively designed to meet our needs in precision and power; in man-power numerically increased to a level commensurate with the opportunities and responsibilities implicit in such programmes.

CALCULATION OF ORBITAL ELEMENTS WITH THE AID OF AN ELECTRONIC COMPUTER

By John F. Heard and Donald A. MacRae

Introduction

During the first half of this century, a good deal of effort went into the determination of orbital elements of spectrographic binaries. More recently orbit determination has received less attention, largely perhaps because the labour involved remains arduous and is of a routine nature. Yet the needs for new data are increasing rather than diminishing. We may cite four such needs:

(a) the continuing need for orbital elements of new spectrographic binaries to provide material for statistical studies involving the origin and development of binary systems,

(b) the need for spectrographic elements to complement photometric studies of the eclipsing binaries which are receiving so much attention during this decade,

(c) the need for more detailed study of special features of spectrographic binaries such as gas streaming, along the lines so often stressed by Struve,

(d) the need for redetermination—in some cases at regular intervals—of the elements of binaries in order to study changes in the orbits.

Any new approach to the task of orbit determination which will reawaken interest or shorten the labour should therefore be investigated.

The determination of the elements of a spectrographic binary involves three things: the observing, the plate measuring and the computing. To a degree, they go hand-in-hand in that, unless the period is known beforehand, the measuring must be kept up to date as the plates are obtained in order to determine the period, and then the observing must be scheduled at such times as to build up an even distribution of points on the velocity curve. Then it is a simple matter, by one of several graphical methods, to determine approximate, or "preliminary", elements. Up to this point, there appears to be no possible short-cut, no substitute for close and careful attention to the job.

With the approximate orbit determined, the observer must decide whether or not to proceed with a least-squares solution for the improvement of the preliminary elements. Some enjoy this arithmetical discipline, others find it tedious; it must be admitted that even with the help of a

good desk computer, it is very time-consuming. The results, too, are often a little disappointing in that the least-squares corrections to the preliminary elements may be trivial in amount, frequently well within the mean errors. It was for these reasons that some observers advocated omitting least-squares solutions, especially when the accuracy of the observations is poor. They overlooked, however, an important point, namely that it is only a least-squares solution which can provide an estimate of the mean errors of the elements—an important consideration if one is to combine or compare spectrographic and photometric elements or to look for small changes in the orbit with time. Observers who do make least-squares solutions frequently group their observations into about twenty normal points in order to reduce the labour of obtaining the normal equations. This practice can lead to a waste of observational material if it is not done with care, and it is hardly ever justified if the period is included in the solution, since points in different cycles should not then be combined. As to whether or not the period *should* be included in the solution, it is often tempting to regard the period as already sufficiently well determined by virtue of a large number of cycles covered by the observations. Sometimes this is not justified, but omission of the period from the solution makes a very appreciable reduction in the labour.

In short, it seems fair to say that spectrographic binary observers, if not loath to undertake least-squares solutions in the first place, are at least eager to make some compromises to reduce the computing to a reasonable amount. If the problem could be programmed for an electronic computer it is clear that both the tedium and the strain on the conscience would be eliminated. One of us (D. A. MacRae) therefore initiated this project, taking advantage of a generous allotment of time on FERUT made available by the Computation Centre of the University of Toronto. As the methods were developed, we tested them on a number of spectrographic binaries which were being made ready for solution.

FERUT and Transcode

FERUT (Ferranti, University of Toronto) is an automatic digital computer of the stored-programme type. *Transcode* is an automatic coding system which has been written for FERUT, making it possible to input from a punched tape a set of relatively simple instructions called a programme, which are then automatically translated by the computer into the more detailed form required for the actual calculation.

The use of the computer is simplified by the availability of a number

of library programmes which have been written to carry out standard mathematical processes. These can be read into storage locations at the beginning of the calculation and then called upon at will by a single instruction.

Transcode is well adapted to make generous and efficient use of the procedure known as looping, that is, the performance of some sequence of instructions a prescribed number of times before proceeding to the next stage of the calculation. This is precisely the type of computing that is required in least-squares solutions.

The General Nature of the Problem

Let us review briefly the usual procedure in the determination of the elements of a spectrographic binary orbit.

The observed quantities are the times of observation and the radial velocities. Given a sufficient number of observations, we proceed as follows:

1. Determine the period by trial and error. If observations are available over a long period of time, the period may often be determined at this point with sufficient precision. Otherwise it should be included in the least-squares solution.
2. Draw a velocity curve, and, by one of several graphical methods, determine the following additional preliminary elements:

   γ, the velocity of the centre of gravity,
   K, the amplitude of the velocity variation,
   e, the eccentricity of the orbit,
   ω, the longitude of periastron,
   T, the time of periastron passage.

   It frequently happens that the orbit is so nearly circular that one can do no better than to take $e = 0$ for the preliminary solution. In this case ω is meaningless and so is T. And in the event that e is small, say less than 0.1, ω and T are poorly determined.
3. Introducing differential corrections to the elements ($\delta\gamma$, δK, etc.), choose one of several forms of a linear equation of condition relating functions of these to the deviations of the observed from the computed velocities. The functions of the differential corrections are the unknowns in the least-squares solution (as many as there are differential corrections) and the coefficients of the unknowns are functions of the preliminary elements and the phases of the observations. These coefficients must be determined for each observation—or for each normal point if the observations are grouped. Grouping will lead to different

weights for the equations of condition—as may also the differences in the observations themselves.

The choice of the form of the equation of condition is largely, but not altogether, a matter of personal preference for a particular means of computing the coefficients. Schlesinger contributed important tables to facilitate this part of the work. More recently, Sterne (1941) discussed the advantages of various forms of the equation of condition and suggested two somewhat different from that of Schlesinger for use in different circumstances:

form 1, for use when the eccentricity is sensibly different from zero,

form 2, for use when the eccentricity is so small that it may be taken as zero in the preliminary solution.

Both of these forms introduce an element T_0, the "epoch of the mean longitude". T_0 is related to T, the time of periastron passage by the relation,

$$T_0 = T - \frac{\omega}{\mu}$$

where ω is the longitude of periastron and μ is the mean daily motion. The advantages which Sterne claims are that T_0 retains a meaning when $e = 0$ (it is then the moment of passage through the ascending node) and that it is more determinate than is T when e is small.

4. Form the normal equations from the equations of condition. This stage is particularly laborious by desk calculator if the equations of condition are many.
5. Solve the normal equations for the unknowns. The method of Gauss was used almost exclusively until fairly recently when matrix methods have been gaining favour. Either method is laborious by desk calculator.
6. Compute the mean error of an observation of unit weight, the weights of the unknowns, and thus the mean errors of the unknowns. In the method of Gauss, the computation of the weights of the unknowns is additional; in matrix methods the weights appear as the diagonal terms in the inverse matrix in the course of the solution of the normal equations.
7. Compute the differential corrections to the preliminary elements and thus the new elements and their mean errors. These are relatively easy computations.

The Machine Methods

The programmes which have been written for FERUT follow this pattern:

The input data-tapes consist of certain constants including the preliminary elements and some simple functions of them, the phases of the observations (if the preliminary eccentricity is zero), the true anomalies (if the eccentricity is not zero), the time intervals since T_0 (if the period is included in the solution) and finally the observed velocities. It takes a stenographer a matter of an hour or so to punch these data for a particular binary. Following the data tape the library programme tapes and the main programme tapes of instructions are read into the machine in sections as needed. Read-in time is a relatively small fraction of the total machine time required.

FERUT computes first the coefficients of the equations of condition and then uses them to form the normal equations which are printed out and also stored on the memory drums of the machine. During this stage, there is also computed and printed the weighted sum of the squares of the residuals (Σpv^2) for the preliminary orbit. In the next stage of computation the inverse matrix of the normal equations is obtained and printed out. Then the values of the unknowns are calculated and printed out along with their mean errors and also the Σpv^2 for the final orbit and the mean error of an observation of unit weight. In the final stage FERUT computes and prints out the corrected elements and their mean errors and the velocities from these elements for twenty-one evenly spaced points in the cycle. In the course of the computation there are repetitions, checks of the symmetry of matrices and formations of check sums. These checks are all shown in the output, and our experience has been that they entirely safeguard against the possibility of undetected errors.

The whole routine can be carried out without an understanding of the programme by anyone competent to operate FERUT. It can be done without interruption in a time ranging from 30 to 45 minutes, but it can be interrupted at several stages if an error is detected or if insufficient machine time is available to complete the routine in a single session.

The Programmes

The same routine is not suitable for all cases, and the following table shows the programmes that have been written and tested to the time of writing.

It is proposed to continue development of these methods, re-writing the programmes so as to remove the limitation of 40 normal points for the cases of five unknowns, and also eliminate the need for including true anomaly in the input for the cases of non-zero preliminary eccentricity, and writing a programme for the case of a double-lined binary.

Programme for	Five unknowns	Five unknowns	Six unknowns
Preliminary eccentricity	zero	non-zero	non-zero or zero
Limitation	40 observations or normal points	40 observations or normal points	No limit
Form of equation of condition	Sterne's no. 2	Sterne's no. 1	Sterne's no. 1 modified
Input data	a) The square roots of the weights of the observations or normal points	a) The square roots of the weights	a) The weights
		b) True anomalies*	b) True anomalies*
	b) L(= phase from T_0)		c) Time since T_0
	c) Observed velocities	c) Observed velocities	d) Observed velocities
	d) Preliminary elements	d) Preliminary elements	e) Preliminary elements

*The true anomalies are easily read from the tables of Schlesinger and Udick (1912). There is but a single interpolation if the preliminary eccentricity is chosen with these tables in mind.

Conclusion

The programmes for least-squares adjustment of the elements of spectrographic binaries which have been described have proved successful in that they reduce computing time to an almost negligible amount and thereby eliminate the need to introduce short-cuts which may not be justified. For example, a solution for the binary H.D. 30353 involving corrections to six elements from 61 observational equations was accomplished in approximately 40 minutes of machine time.

The methods described here offer interesting possibilities for the revision of orbits already published as well as for the computing of orbits from new data. The writers could undertake computation for other observers, and they invite correspondence from any who have data which they have already worked up as far as the preliminary orbit.

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VERY CLOSE AND VERY WIDE DOUBLE STARS

By G. Van Biesbroeck

Our knowledge of stellar masses is based on a small number of binary orbits. In order to consolidate that information which is basic in all work on stellar constitution there is still a great need of well-determined binary orbits. The most promising approach to fill this need is through the observation of rapidly moving pairs which, combined with parallax and mass-ratio determinations, will yield masses in a relatively small number of years. Rapidly moving pairs means close pairs that can be measured only by micrometric or interferometric methods. It means also pairs that require good-sized telescopes. My experience with the 82-inch reflector of the McDonald Observatory has shown that reflectors are fully as efficient as refractors for that type of work and that the larger apertures of the former give them a decided advantage in separating close pairs. The systematic Lick survey of the northern hemisphere brought forth a large number of such close pairs. Even more have been added in the recently completed southern survey. There is a high percentage of these stars in which, in spite of a half-century of observations, there is still great uncertainty about the elements.

As an illustration let us consider the bright star β Leonis Minoris, which Hussey found in 1904 to consist of two components of magnitudes 4.0 and 6.5 separated by 0".4. Figure 1 shows that after 1906 the companion closed in until it was lost after 1912 in the glare of the bright star. Not until 1927 was it picked up again, still in the third quadrant. Further observations showed that the separation increased to 0".5 in the early forties, then decreased again. From the recurrence of the angles a period of some 40 years is indicated. An orbit was even computed from this insufficient material but since all measures were in a narrow sector up to 1954, the elements remained completely undetermined. This spring with the 82-inch reflector the companion was located for the first time in the first quadrant at a separation of 0".11. A critical phase has been reached and measures in the immediate future will lift the doubt about the elements. No matter how much the star were observed in the third quadrant the elements could never be determined. A few observations at the present crucial stage are all that is needed. This is a good example of the importance of using telescopes of large aperture on close pairs.

In the star just considered the large difference in magnitude eliminates

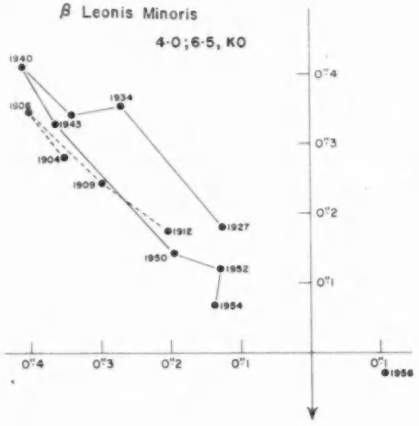

FIG. 1—Separation (in seconds of arc) of the two components of β Leonis Minoris (Hu 879, A.D.S. 7780) covering the period 1904 to 1956.

the uncertainty of the quadrant. There is another group of binaries in which there is the additional difficulty that the two components are nearly alike in brightness. After the pair has closed in and opened up again, there remains the question whether there has been a reversal of quadrant. There is unfortunately a great number of pairs of this type. A classical example is ξ Scorpii, a pair which was noted by W. Herschel. The usable measures begin in 1825 and the course of the star is shown in figure 2, reproduced from T. J. J. See's volume published in 1896. Both Schorr and See deduced orbits of small eccentricity with a period of nearly a century. After the turn of this century larger telescopes were available and the star was well followed near conjunction. The open circles marked 2, 3, 4, 5, 7 show how the companion moved from 1902 to 1907. Instead of a nearly circular orbit we find that we are dealing with an eccentricity of 0.75 and a period of 44 years. The observations in the lower half of the figure should be changed by 180° to conform to the recent measures. It took nearly a century to establish that, because the separation at the 1865 conjunction was too small to be detectable by the telescopes used at that time. The new orbit reduces the masses to nearly one-half of the earlier determination.

Many stars of this type are found among the Aitken and Hussey pairs and still more among the southern pairs observed by Van den Bos,

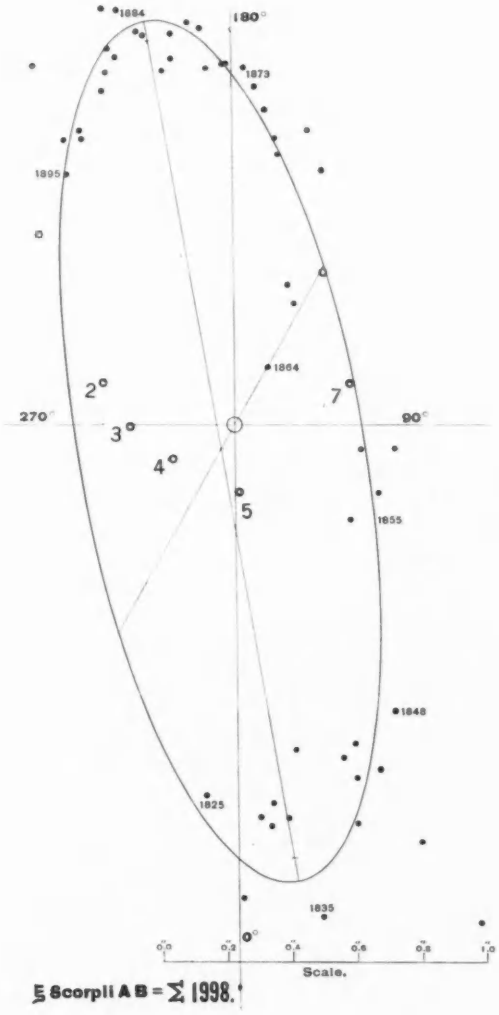

FIG. 2–The orbit of ξ Scorpii, as determined and drawn by See (1896), with more recent observations (1902–1907) indicated by numbers 2, 3, 4, 5, 7.

Rossiter, *et al.*, hence the importance of devoting some of the time available with instruments of large aperture to the elucidation of such cases of quadrant-ambiguity. If smaller instruments only are used the same arc will be covered again and again and the critical phase of close approach, which alone can give a clue to the real situation, can never be reached.

It is of interest to compare large double-star orbits, on the average,

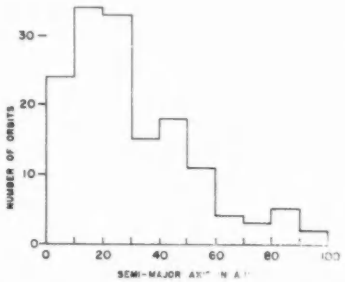

FIG. 3—The length of the semi-major axis in A.U. is shown plotted against the number of orbits, for a total of 163 orbits.

with planetary orbits in the solar system. Figure 3 shows the result of a count covering 163 fairly well-determined orbits of double stars for which sufficiently reliable parallaxes are available. It is at once evident that the greatest number of binary orbits have sizes of the order of 30 astronomical units. On the smaller side the information is evidently incomplete because many such pairs are too close for visual detection until we run into the spectroscopic binaries. The decrease in number is, however, well marked after a major axis of 100 A.U. and naturally the question arises to what separations do exist in binaries. When the separation increases we soon run into periods of thousands of years. Here the photographic observations are enormously superior to the micrometric ones but centuries will have to go by before something definite about orbital motion can be established. When the separation gets very large there is a better clue than slow orbital motion to establish the physical connection, namely common proper motion. For widely separated binaries we therefore turn to the list of proper motions such as Luyten's recent LFT catalogue covering the known stars with proper motions exceeding 0".5 a year. The following table lists by increasing separations the widest binaries known at present for which reliable parallaxes have been determined, beginning with LFT 1178 where the projected separation is 7500 A.U.

WIDE BINARIES

LFT		Mag.	Proper Motion	Separation (″ of arc)	Parallax	Separation (A.U.)	
1178	−15°4041–2	9.8 10.5	3″.68	301	0″.040	7500	GC
	γ Lep	3.8 17.0	0.46	1130	0.122	9200	VB
253	+61°513	7.4 13.3	1.00	266	0.027	9900	VM
1110	α Cen C	0.3 13.2	3.69	8000	0.751	10600	L
1361	26 Dra	5.8 11.2	0.57	740	0.064	11600	VM
549	−33°4113	5.4 18	1.71	870	0.057	15300	VB
1416	+8°3689–92	8.6 9.3	0.53	610	0.032	19000	Ci
	β Ret	3.8 9.0	0.33	1480	0.042	35000	L
	−32°6181	9.3 11.2	0.45	4600	0.104	44000	L

The inclination of the line joining the components is unknown hence the separations are minimum values. The second star, γ Leporis, was found to have a 17th magnitude companion some 19′ away. This was noted in the course of a systematic search for intrinsically faint stars which has been going on for some years at the McDonald Observatory. (See figures 4 and 5.) In the case of LFT 1416 the parallax is small so that the separation is uncertain, but the order of magnitude must be right.

FIG. 4—The field of — 33°4113. The position of the faint companion, whose image is clearly seen on the original photograph, is indicated by the white arrow. North is at the left.

FIG. 5—The field of γ Leporis. The position of the faint companion, whose image is clearly seen on the original photograph, is indicated by the white arrow. North is at the top.

The largest separation is for Luyten's pair — 32°6181 where the angular distance of one and one-quarter degrees can be safely converted with a sizable parallax into 44000 A.U. This is about one-eighth of the distance of α Centauri and if the masses are of the same order as the sun the period must be some 10 million years. There is evidently no definite limit to the separation of binary stars. At some of the large distances listed, perturbing influences of other stars may become appreciable until we come to star clusters and associations.

Comments made by S. S. HUANG:

The binding energy of a very wide binary system is so small that it can be easily disrupted by a stellar encounter. Hence some of the wide binaries that Dr. Van Biesbroeck has investigated may turn out to be unstable in the sense that they will dissociate, as a result of "thermal motion" of the neighbouring stars, before they are able to complete a few revolutions. As a rough estimate, we can consider as unstable those binaries in which the separation of the two component stars exceeds, say, one-half of the mean interstellar distance. Therefore Dr. Van Biesbroeck's result strongly indicates that those wide binary stars must have been formed recently, in this case by capture in a collision of three or more bodies. Consequently, we may suggest that for very wide binary systems a kind of equilibrium exists such that formation of binaries balances their dissociation in a manner not unlike the ionization of a gas at a definite temperature and pressure. However, such a statistical consideration should not be applied to the close binaries, because the chance of close stellar encounters is too small to have established an equilibrium state in the solar neighbourhood.

A RECONSTRUCTION OF THE MAIN SEQUENCE FROM DIFFERENTIAL OBSERVATIONS OF BINARIES

By Willem J. Luyten

In this paper two rather divergent topics will be discussed: (A) the establishment of the H-R diagram from binary star data alone, and (B) an analysis of the data on white-red dwarf binaries with proposals for future observations aimed at determining the masses of white dwarfs.

(A) Some thirty years ago (1923) Lundmark and I proposed a method for the determination of the relation between stellar luminosities and spectra using only the differences in apparent magnitude and spectral class for the components of binary stars. There has been a great increase in observational data since that time—in numbers of stars available as well as in the accuracy of the data—and it would seem worth while to repeat the analysis with the newer data. Owing to the rather short time available, however, this analysis had to be restricted to the following data: (a) all those stars from Wallenquist's *Catalogue of Magnitude Differences for Double Stars* (1954) for which spectra could also be found, and (b) double stars with large proper motion—generally rather wide pairs.

Again, owing to the short time available, it was not possible to comb the literature for accurate spectral classifications for Wallenquist's stars and we had to be satisfied with the classification we could find for them in Aitken's *General Catalogue*. Hence only 273 stars out of the more than one thousand in Wallenquist's catalogue could be used; moreover, most of the spectra are classified on the old Henry Draper system. The magnitude differences, therefore, are more accurate than the differences in spectral class. For the stars of large proper motion the situation is almost reversed, for here the spectral types are mostly modern, and accurate, but the magnitude differences are often only crude estimates, mostly my own.

In the analysis of these data I have tried to proceed as objectively as possible but no matter how much one can pretend to be objective it seems reasonably certain that the main features of the H-R diagram are constantly in the back of the mind. Consequently one can only hope that such bias does not affect the conclusions too seriously. Two further, purely practical, difficulties may be mentioned. (1) While the magnitude scale is uniform over the entire range of data we have as yet no spectral classification that can be linearly expressed in terms of a real physical

parameter such as log T. We simply have to make do with the existing, rather arbitrary spectral designations and for this reason it was necessary to subdivide the material into rather narrow spectral-class groups. (2) Since the crucial quantity is the quotient $\Delta m/\Delta S$ it is evident that, especially for the older, more uncertain spectral classes we can expect trouble when the two components differ very little in spectral class. Arbitrarily, therefore, I have treated all doubles with $\Delta S \leqslant 0.2$ spectral class units separately from those with $\Delta S \geqslant 0.3$ units.

Wallenquist's material was divided into six groups, according to whether the primaries were of spectral class B, A, F, etc. For each group plots of Δm against ΔS were made; in all cases one or more loci became clearly indicated and average numerical values were then obtained by taking $\Sigma\Delta m/\Sigma\Delta S$. The results for all groups are summarized below; in each case the rates found were used to correct the average spectral class of each group to the nearest whole tenth. In the A group there were 43 pairs with spectral class differences 0.2 or less; these divided themselves rather neatly into two groups, $\overline{\Delta m} = 1.24$ (17 stars) and $\overline{\Delta m} = 0.26$ (26 stars). The former probably represents the difference between "giants" and "main-sequence" stars of nearly the same spectral class at about A3 while the latter represents no more than the observational inaccuracies in spectral classification of nearly identical stars. For the F stars we find similarly $\overline{\Delta m} = 1.15$ (9 stars) and $\overline{\Delta m} = 0.22$ (18 stars) at about F3. For the G and K stars these groups were not sufficiently numerous to lead to significant results.

Mean Spectral Class			
Primary	Secondary	$\overline{\Delta m}$	Number of Stars
B4	B8	2.09	36
B7	G6	1.42	3
A3	F9	2.56	15
A3	G4	0.28	6
F3	G5	2.21	41
F3	A5	1.10	16
G3	K2	1.77	14
G5	K0	0.27	4
G5	F1	1.34	12
K2	K7	1.84	11
K2	F0	1.50	18
M2	M4	0.78	4
M2	M1	3.38	5

Applying the same analysis to the pairs with large proper motion, among which we should expect an overwhelming proportion of main sequence stars and hence usually only a single value for Δm we find:

Spectra	Δm per whole unit of spectral class	Number of Stars
A–F	1.90	6
F–G	2.27	16
G–K	2.48	20
K	4.20	28
K–M	4.22	31
M	8.30	24

In addition we have three pairs (α Gem, ι UMa, α Leo) where for mean spectral classes A1 to K7 we find $\Delta m = 7.1$, three others (α Tau, α Aur, ϵ Cyg) with G8 to M4 and $\Delta m = 12.1$, and hence obviously giant primaries with main-sequence secondaries, while finally there are three pairs with degenerate companions yielding F9 and DA with $\Delta m = 7.5$.

It is evident that a certain amount of adjustment needs to be made since the values given above do not all agree exactly. If such adjustment is made and if further it is arbitrarily assumed that M(vis.) for A0 is 0.0 we find:

	Wallenquist		Large Proper Motion Stars	
	Main Sequence	Giants	Main Sequence	Giants and White Dwarfs
B0	−5.18		—	
A0	0.00		0.00	
F0	+1.80	A3 −0.70	+ 1.50	
G0	+3.64	F3 −0.20	+ 3.60	
K0	+5.61	G5 +0.44	+ 5.70	G8 − 0.4
M0	+8.91	K0 +0.60	+ 9.10	DA +10.9
M5		M2 −1.40	+12.4	
M8			+14.7	

Assuming for the sun, m(vis.) = − 26.9 and G2, we then derive a correction of +0.75 to the above data; combining them into one system we obtain the values shown below. The corresponding values now generally accepted for stars on the main sequence as determined from trigonometric and statistical parallaxes are given in the third column.

	Final	Trig. Parallax	Giants and White Dwarfs
B0	− 4.43	− 4.	
A0	+ 0.75	+ 0.7	
F0	+ 2.40	+ 2.7	A3 + 0.05
G0	+ 4.35	+ 4.4	F3 + 0.55
K0	+ 6.40	+ 5.8	G6 + 0.95
M0	+ 9.80	+ 8.7	K0 + 1.25
M5	+13.2	+11.7	M2 − 0.65
M8	+15.4	+14.0	DA +11.6

From B0 to G0 the agreement is almost better than could have been hoped; beyond G0 it is possible that the occurrence of even an occasional subdwarf or intermediate among the secondaries has made the slope too steep. If the entire material could be rediscussed with the more refined classification on the MKK system now available, it would seem that not only could still closer agreement be reached but eventually a large part of the "fine structure" of the H-R diagram might be recognized and reconstructed. Finally, it may be emphasized that the results given here have been obtained without the use of a single trigonometric, spectroscopic, dynamical, cluster, or statistical parallax of any kind and that, because of their differential nature, these results are influenced only slightly, if at all, by obscuration in space. If it could be shown in this way that the H-R diagram is indeed a unique aggregation of loci these same results could play an important role in establishing the distance scale of the universe.

(B) Among binary stars of all kinds perhaps those which contain a white dwarf component may be cited for special attention. Orbits are known for only three such systems, but two of the white dwarfs involved may be non-typical, and the mass of the third is uncertain. If we are to obtain further information on the masses of white dwarfs we necessarily have to fall back on statistical conclusions obtained from wide pairs with barely discernible orbital motion. From the data now available we find twenty-nine such pairs while six more are suspected. Calculating probable orbital motions by statistical means (1930, 1933) we further find that for fourteen of these the expected orbital motions will lie between 0″.0004 and 0″.0045 annually; these pairs are obviously of little interest now, as sufficiently accurate determination of their orbital motion will be a very long-term project. The other fifteen pairs are more promising, since their expected orbital motions range from 0″.005 to 0″.030 annually.

Detailed information on all these stars will soon be published elsewhere; suffice it for the moment to urge all astronomers who have long-focus astrometric refractors or large reflectors available, to begin observations of these stars as soon as possible. If second epoch plates are then taken a few decades from now, preliminary values for these orbital motions could be obtained with sufficient accuracy to permit one to make at least an estimate of the masses of white dwarfs.

Orbital motions calculated by statistical formulae rest on a number of assumptions. It would seem wise, therefore, that as control objects at least an equal number of similar faint, wide pairs be observed for which we are certain that both components are main-sequence red dwarfs. These control pairs can be selected easily from among the 832 LDS pairs

published for the southern hemisphere (1941) and the several hundred similar pairs found in the northern hemisphere blink survey.

Finally, special attention may be called to two objects, LDS 275 and HZ 9. The former is still the only known pair of white dwarfs for which, in addition, a preliminary value for the orbital motion is available. This motion is almost entirely radial, and amounts to 0".034 annually. While it is much too early to derive an orbit, an inclination close to 90° is suggested. Values of $P = 700$ years, $a = 5''.3$ and a parallax of 0".06 seem to be of the right order of magnitude; these would yield a value of 1.4 for the combined mass.

In the spectrum of HZ 9 = L1239 — 16, Greenstein (1956) has found emission lines of Ca II which, moreover, indicate a velocity variation of more than 250 km./sec. All this suggests the fascinating possibility that this star may be a white-red dwarf eclipsing binary, a suggestion which is enhanced by the fact that although the star was found in Zwicky and Humason's blue-star survey it is not really a blue star since its *B-V* colour index is + 0.33. (It may have been in eclipse at the time Humason and Zwicky's yellow plate was taken.) If the star is an eclipsing binary, a period of the order of half a day, and a primary minimum lasting around half an hour are indicated. Obviously this is an object well worth following up photometrically and spectroscopically.

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STELLAR MASSES AND THE MASS-LUMINOSITY RELATION

By K. Aa. Strand

The present paper is an attempt to summarize the data which are presently available and those which are further needed, with regard to obtaining a list of representative masses and luminosities for components of visual binaries from which the empirical mass-luminosity relation and the dispersions within the diagram can be established within a certain degree of precision.

The 37 binary systems listed in the table fall within the category of systems for which the orbits can be considered as reliable, or at least the error in $3 \log a'' - 2 \log P$ to be small, and which have parallaxes in excess of $0''.050$. Since the percentage error in the parallax is tripled in the mass determination and since the combined systematic and accidental error inherent in the trigonometric parallaxes in no case is less than $\pm 0''.005$ (p.e.), systems with trigonometric parallaxes smaller than $0''.050$ will provide stellar masses with probable errors larger than 30 per cent. and accordingly contribute little to the problems under consideration.

Because of limited space, no references have been given to the sources from which the material has been collected for Table I, neither is any detailed discussion given of the accuracy of the listed quantities, except by the number following a preceding quantity. While (1) indicates that the quantity is determined with satisfactory accuracy, (2) shows that more accurate data are desirable, and (3) that further improvements in the data are definitely needed.

Examining the table as a whole, we find that the data which are especially needed to complete and further improve on our observational data for the establishment of the empirical mass-luminosity relation are:

1. Stellar parallaxes and mass ratios from extended photographic series.

2. Photoelectric magnitudes on an established magnitude-colour system. For the close pairs their combined magnitudes and their magnitude differences by the Kuiper (1933) or Muller (1948) methods.

3. Spectral classification on the Yerkes system and/or colours in the *U,B,V* system.

4. Radial-velocity determinations to establish the orbits in the line of sight for checking on possible systematic errors in the trigonometric parallaxes.

While certain of the data listed above will not be easy to obtain, such as the spectra or colours of the fainter component in the close systems, it

TABLE I

DATA FOR THIRTY-SEVEN VISUAL BINARY SYSTEMS

ADS	Star	α 1950	δ 1950	Orbit P	Orbit a	Qual.	π	Qual.	M/R	Qual.	Mag.	Qual.	Δm	Qual.	Spectra
61	Σ 3062	$0^h03^m.5$	+58° 09′	106ʸ.8	1″.43	(3)	″.048	(2)	.26	(2)	6.10	(2)	.73	(2)	G3, –
490	13 Cet	0 32.7	− 3 52	6.9	.24	(2)	.058	(2)	.44	(2)	5.24	(2)	.6	(3)	F7, –
520	β 395	0 34.7	−25 02	25.	.67	(1)	.070	(2)	—		5.71	(2)	.1	(3)	K0, –
671	η Cas	0 46.1	+57 33	480.	11.99	(2)	.170	(1)	.39	(1)	3.45	(1)	3.79	(1)	G0 V, K5+
—	L 726–8	1 36.4	−18 13	32.	1.65	(3)	.381	(2)	—		11.9	(3)	.5	(3)	M6e, M6e
1865	A 2329	2 25.1	+ 4 12	25.	.55	(3)	.070	(2)	—		8.7	(3)	.2	(3)	K5, –
—	ε Cet	2 37.2	−12 05	1.59	.12	(3)	.059	(3)	—		4.86	(1)	.0	(3)	F4
△ 3093	o_2 Eri (BC)	4 13.2	− 7 41	248.	6.9	(3)	.201	(2)	.31	(2)	9.29	(1)	1.68	(1)	B9, M5e
△ 3841	α Aur	5 13.0	+45 57	.29	.056	(1)	.074	(1)	.53	(2)	.05	(1)	.15	(2)	G5 III, G0 III
—	Ross 614	6 26.8	− 2 46	16.5	.98	(2)	.251	(1)	.36	(2)	10.9	(3)	3.5	(3)	M6, –
△ 5423	α CMa	6 43.0	−16 39	49.9	7.62	(1)	.377	(1)	.33	(2)	−1.42	(1)	10.0	(3)	A1 V, A5
△ 6251	α CMi	7 36.8	+ 5 22	40.6	4.55	(1)	.287	(1)	.27	(1)	.37	(1)	10.3	(3)	F5 IV, –
△ 6420	9 Pup	7 49.5	−13 46	23.2	.58	(1)	.060	(2)	.50	(2)	5.27	(2)	.7	(3)	G1 V, –
6650	ζ Can (AB)	8 09.3	+17 48	59.7	.88	(2)	.047	(2)	.47	(3)	4.95	(2)	.28	(2)	F7, –
—	10 U Ma	8 57.6	+42 00	20.8	.61	(3)	.073	(3)	.40	(2)	3.96	(1)	2.06	(2)	F5 V, –
7284	Σ 3121	9 14.9	+28 47	34.2	.66	(2)	.059	(3)	—		7.26	(2)	.1	(3)	K0
△ —	ψ Vel	9 28.7	−40 15	34.1	.92	(2)	.062	(2)	.49	(2)	3.56	(1)	.9	(3)	F0, –
8119	ξ U Ma	11 15.6	+31 49	59.9	2.54	(1)	.126	(3)	.50	(3)	3.80	(1)	.45	(2)	G0 V, G0 V
△ 8630	γ Vir	12 39.1	− 1 11	171.	3.75	(2)	.089	(2)	.49	(3)	2.76	(1)	.02	(2)	F0 V, F0 V
8804	42 Com	13 07.6	+17 47	25.9	.66	(1)	.057	(2)	—		4.47	(2)	.00	(2)	F5, F5
△ 9031	Σ 1785	13 46.8	+27 14	155.	2.42	(2)	.075	(2)	.49	(2)	7.26	(2)	.49	(2)	K6, K6
△ —	α Cen	14 36.6	−60 38	80.1	17.59	(1)	.761	(1)	.46	(2)	−.30	(1)	1.24	(1)	G4, K6
△ 9413	ξ Boo	14 49.1	+19 19	150.	4.88	(2)	.145	(2)	.46	(2)	4.57	(1)	2.15	(2)	G8 V, K5
9617	η Cr B	15 21.1	+30 28	41.6	.84	(1)	.065	(3)	.48	(2)	5.05	(2)	.25	(2)	G2 V, G2 V
△10157	ζ Her	16 39.4	+31 41	34.4	1.37	(1)	.103	(1)	.40	(1)	2.81	(1)	2.55	(2)	G0 IV, K0 V
—	−8° 4352	16 52.8	− 8 14	1.71	.20	(2)	.152	(1)	—		9.2	(3)	.1	(3)	M4, M4
△ —	Fu 46	17 10.6	+45 47	14.0	.71	(2)	.147	(2)	.44	(2)	9.4	(3)	.4	(3)	M3, –
△ —	Melb 4 (AB)	17 15.5	−34 56	42.1	1.82	(1)	.142	(1)	.41	(2)	5.9	(3)	1.5	(3)	K5, –
△10660	26 Dra	17 34.5	+61 55	74.2	1.50	(1)	.073	(1)	.45	(1)	5.22	(1)	2.61	(2)	G0 V, –
10786	μ Her (BC)	17 44.5	+27 46	43.0	1.29	(1)	.108	(3)	—		10.2	(3)	.5	(3)	M5, –
△11046	70 Oph	18 02.9	+ 2 32	87.8	4.55	(1)	.199	(1)	.42	(1)	4.00	(1)	1.75	(2)	K1 V, K5 V
△11077	99 Her	18 05.1	+30 33	56.4	1.03	(1)	.058	(1)	.36	(2)	5.04	(1)	3.4	(3)	F7 V, –
△11871	β 648	18 55.2	+32 50	59.8	1.26	(1)	.059	(2)	.47	(2)	5.21	(2)	1.8	(3)	G0, –
△14773	δ Equ	21 12.0	+ 9 48	5.70	.26	(2)	.056	(2)	.49	(2)	4.61	(2)	.0	(3)	F8V
14787	τ Cyg	21 12.8	+37 49	49.8	.85	(3)	.043	(3)	.42	(2)	3.82	(2)	2.82	(2)	F2 III, –
△15972	Kr 60	22 26.3	+57 27	44.5	2.39	(1)	.253	(1)	.38	(1)	9.59	(1)	1.58	(2)	M4+, M6
△17175	85 Peg	23 59.5	+26 49	26.3	.83	(1)	.080	(1)	.49	(1)	5.75	(1)	3.04	(2)	G3V, –

should be realized that there is little prospect of increasing, within a foreseeable future, the systems belonging to the category in Table I, except through the discovery of short-period binaries among the near-by stars from: 1. Proper motion surveys; 2. Resolution of astrometric binaries; 3. Resolution of "single" stars and spectroscopic binaries by means of the interferometer.

For 25 of the systems listed in Table I (marked with Δ) the available data are known with sufficient accuracy to establish the logarithms of the masses ($\log \mu$) of the components with a probable error of less than ± 0.13 (errors in mass less than 30 per cent.). These data are plotted in figure 1 against their absolute bolometric magnitudes using the bolometric corrections by Kuiper (1938). Because of lack of spectral information on which to base the bolometric corrections, the data for only fifteen of the secondary components were included in the plot.

Accordingly, figure 1 contains the plotted positions of absolute bolometric magnitudes and masses of forty visual binary components, which define a sequence ranging from $M_{Bol} = 12.3$ ($\log \mu = -1.4$) to $M_{Bol} = -0.3$ ($\log \mu = +0.4$) if we exclude the data for the three white dwarfs which show the well-known deviation. The data shown in the

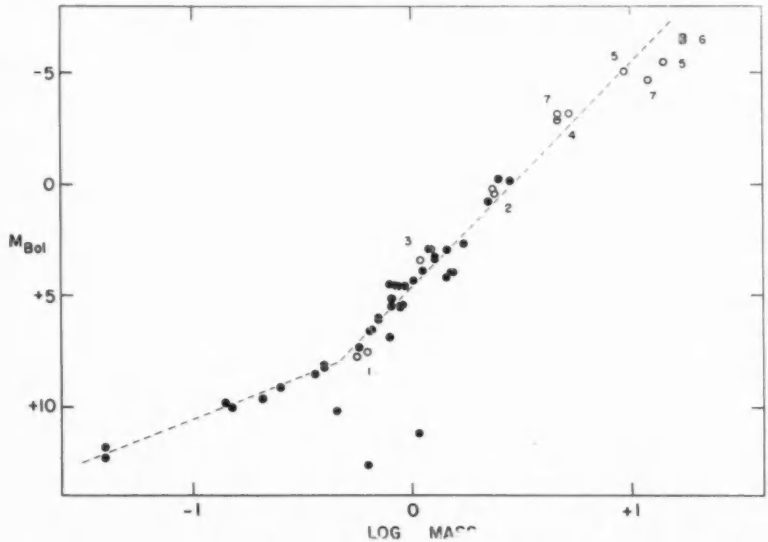

FIG. 1—The mass-luminosity relation for components of visual binaries (filled circles) and spectroscopic binaries (open circles). The two broken lines were computed on the basis of a linear relationship between absolute bolometric magnitude (M_{Bol}) and the logarithm of the mass.

diagram are essentially those listed by Strand and Hall (1954) with minor revisions resulting from improved observations which have become available since, and adding the results for five red dwarf components (Ross 614A and both components of $-8°4352$ and L726–8).

Assuming a linear relationship between the two quantities under discussion we find:

For stars between $M_{Bol} + 7.5$ and -0.3:

$$M_{Bol} = 4.62 - 10.03 \log \mu, \text{ or } L = \mu^{4.0}$$
$$\pm.08 \quad \pm.45 \text{ (p.e.)}$$

For stars fainter than $M_{Bol} = +7.5$:

$$M_{Bol} = 6.76 - 3.80 \log \mu, \text{ or } L = 0.1\,\mu^{1.5}$$
$$\pm.11 \quad \pm.12 \text{ (p.e.)}$$

If we bear in mind that a probable error in $\log \mu = \pm 0.13$ corresponds to an overall probable error of 1.2 in M_{Bol}, when these data are correlated in the diagram, the relation between luminosity and mass appears to be close, since not a single observation deviates by more than one magnitude from the linear relationships established above.

Some of the dispersions shown in the present diagram are undoubtedly due to observational errors while others can be considered as real. As an example of the first kind can be mentioned the components of γ Vir, which previously were considered over-luminous, and now fall close to the mean relationship just established, because of recent improved data for the parallax. The four main-sequence stars falling approximately one magnitude below the established sequence are, in order of increase in mass: ξ Boo B, α Cen B, 70 Oph A and ξ Boo A. While changes in the values for the components of ξ Boo might occur when parallaxes from extended photographic series become available, no significant changes are expected for the two other components just mentioned.

Main-sequence stars lying above the main sequence by approximately one magnitude are, in order of increase in mass, the brighter components of: 13 Cet, 9 Pup and η Cas. For the two first mentioned, the probable errors in absolute magnitude are of the same order as the deviations, while for η Cas the deviation is about twice the probable error.

Among the forty binary components plotted in figure 1 are two subgiants and two giants. While the subgiant ζ Her A shows the well-known deviation from the mass-luminosity diagram, the earlier type subgiant Procyon A and the giant components of Capella show no appreciable deviation.

An extension of the mass-luminosity relation to the stars more luminous than $M_{Bol} = 0$ must be based upon the data available for spectroscopic binaries. From the double-lined spectroscopic binaries which are also

eclipsing binaries, a careful analysis of the photometric orbits and established magnitude-colour system, combined with the spectroscopic data, leads to a direct determination of the mass-luminosity relation beyond the luminosities available from the visual binaries. In this respect the recent work by Popper on selected systems is of particular interest.

In figure 1 are plotted the data for the components of seven spectroscopic binaries which are considered to have adequate data at present. For the early-type stars the bolometric corrections determined by Miss McDonald and Miss Underhill (1952) from model atmospheres were used. The components, identified by numbers in the diagram, are: YY Gem (1) and β Aur (2), both of which have main sequence components; Z Her (3) with subgiant components; U Oph (4), μ Sco (5), and Y Cyg (6) having components which are O-B stars; and finally V 356 Sgr (7) which consists of a main sequence B3 star and an A2 II giant. It is of interest to note that the subgiants Z Her A and ζ Her A occupy identical positions in the diagram.

Although the linear relation established for the visual binaries seems to continue to a stellar mass of approximately 10, a slightly smaller slope for the upper part of the branch would give a better agreement with the four most massive components in the plot. This agrees well with the results by Petrie (1950), who used another approach to finding the mass-luminosity relation for the luminous stars by determining the slope of this relation from the measured magnitude differences and mass ratios, for the components of double-lined spectroscopic binaries.

While the present study has emphasized a close relationship between the masses and the luminosities of the stars, there exist special groups of stars, such as white dwarfs, the W Ursae Majoris stars, the Algol-type variables, as well as individual spectroscopic systems, which seem to violate the mass-luminosity relation. Of particular interest is the mass distribution for the white dwarfs. Several binary systems are known which could provide more data on this problem, but all of these systems are too faint to be observed for parallaxes, mass-ratios and orbital motions with the refractors now used in this field. The solution of these problems will depend upon a reflector suitable for astrometric work on stars fainter than the 13th magnitude.

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CRITICAL STUDIES OF ABSOLUTE DIMENSIONS OF ECLIPSING STARS

By Daniel M. Popper

A. Outline of the Problem

In recent years there have appeared critical analyses of the data providing absolute dimensions of visual binaries (Kuiper, 1938; Strand and Hall, 1954; van de Kamp, 1954). Although very valuable compilations of data and references for eclipsing binaries have been prepared by Kopal (1954, 1955) and by Plaut (1950, 1953), I am not aware of a really critical discussion of these systems. In this paper we discuss the bases upon which such a critical discussion should rest. I wish to make it clear that the best values of the dimensions of eclipsing binaries are not determined here.

This appears to be a particularly suitable time to present such a discussion from several standpoints:

(1) Wide-spread use of photoelectric techniques to obtain light curves of improved accuracy.

(2) The existence of a firmly calibrated relation between colour and spectral type, the *U,B,V* system (Johnson and Morgan, 1953; Morgan, Harris, and Johnson, 1953).

(3) The increased use of grating spectrographs, resulting in availability of the visual and near infra-red regions of the spectrum.

(4) Improved theoretical framework for interpretation of the results, in particular improved models of stellar interiors, as well as new theories of stellar evolution.

The approach adopted in this paper is, first, to decide, somewhat arbitrarily, upon the qualifications a system should meet in order to be considered to provide reliable absolute dimensions; and, second, to classify known systems according to the extent to which they meet the qualifications, or are likely to meet them when improved observational data are obtained. In order to limit the discussion, we shall restrict the analysis to double-lined eclipsing binaries for which the masses and radii may be obtained directly from observational data. As a working list, we shall use Kopal's compilation (1955).

The fundamental quantities to be obtained from the observations for each star are three: mass, $\mathcal{M}$, radius, R, (both in solar units), and colour, $B - V$. Of the related quantities, spectral classification, effective tempera-

ture, and colour, the colour can be obtained most precisely and unambiguously from observation. The observed spectral type should, of course, when obtained *reliably*, be consistent with the observed colour. In order to compare the absolute dimensions of eclipsing binaries with those of visual binaries, it is necessary to know effective temperature (T_e) and the bolometric correction (B.C.) as dependent upon $B - V$. These are auxiliary problems, and cannot be discussed here.

Table I presents in condensed form the crucial and secondary observations needed to determine each of the basic quantities. The accuracy of the derived basic quantity is usually limited by the accuracy of determination of the corresponding "crucial" observational quantity.

TABLE I

Basic Quantity	Crucial Observational Quantity	Also Needed
$\mathcal{M}$	K_1, K_2	P, i, e
R	K_1, K_2, k	P, r, i, e
$B-V$	B, V light curves (U,B,V for reddened stars)	J_1/J_2, spectral types

In this table, K_1 and K_2 are the half-ranges of radial-velocity variation of the two components, k the ratio of the radii, P, i, and e the orbital period, inclination, and eccentricity, respectively, r the radius of a star relative to the distance between centres, and J the surface brightness.

As a guide to our inclusion of a system as "reliable" or not, we should establish rough standards of precision desired for the basic quantities. As one approach to such a problem, we may look at a mass-luminosity plot for visual binaries, and see that a displacement of 0.05 in $\log \mathcal{M}$ or of 0.5 in M, the absolute magnitude, is a significant displacement. Analysis shows that such accuracy requires $K_1 \sin i$ and/or $K_2 \sin i$ to be known to about 4 per cent., r_1 and r_2 (k!) to about 25 per cent., and $B - V$ to about 0.04 mag. for B stars, 0.12 mag. for types A-G8, and 0.22 mag. for types K0-M0. A second approach to this problem is to ask what precision is required to tell whether the star lies on the main sequence to within, say, 0.5 magnitude. In this case, the requirements are that $r \times K$ should be known to within 25 per cent., and $B - V$ to 0.025 mag. for type A0 and earlier and to 0.10 mag. for later types. A third approach, which I have not attempted, is to ask what accuracy in the parameters is required for theoretical problems. In this discussion we have neglected contributions to the uncertainties of the quantities resulting from uncertainties in the bolometric corrections and temperature scale. We shall

use these criteria of accuracy as guides rather than attempt to adhere rigidly to them.

As further guides to the reliability of a system, certain consistency checks should be used. They are more or less useful, depending on the nature of the system.

(1) $B - V$, J_1/J_2, and spectral types should all be consistent.

(2) The brightness ratio, l_1/l_2, from the light curve and from the spectra should be consistent. This check is not useful in practice unless both l_1/l_2 and J_1/J_2 are near unity.

(3) M_{pv} computed from R, T_e, and B.C. should agree with the spectroscopic absolute magnitude if the latter is known with precision. Of these checks the most useful is probably that of $B - V$ against J_1/J_2.

In addition to these quantitative guides to the reliability of a system, two important qualitative decisions need to be made:

(1) Because of theoretical as well as practical difficulties, rectification should not dominate the solution of the light curve. I exclude arbitrarily from a list of reliable systems, for example, all W UMa systems, systems with r_1 or $r_2 \geqslant 0.4$, and systems in which the two components differ greatly in shape unless the circumstances are unusually favourable (e.g. total eclipse).

(2) Can the lines of ***both*** components be ***reliably*** measured in the spectrum?

This last question is really the first one that needs to be answered before deciding whether further analysis of a system is worth while from the standpoint under discussion here. The question is a qualitative one and involves perhaps the most controversial aspect of the whole problem. It can be answered only by inspection of spectrograms or reproductions. This question was raised for a few systems with published orbits of both components by Sahade and Struve (1945). Petrie (1950) also raised the question for a number of systems in connection with his l_1/l_2 measurements. It must be pursued further and examined in the light of photometric data in any critical analysis.

If the answer to question (2) is in the affirmative, and a decision about (1) is favourable, then a system is a good candidate from the special point of view we are taking.

B. Two Special Problems

Having outlined the approach to this subject of a critical analysis of eclipsing systems, I should like to discuss two problems of particular importance, which have already been briefly mentioned.

(1) Reliability of K for the weaker spectrum. I have examined Mount

Wilson and Victoria spectrograms of the great majority of systems on Kopal's list. A (more or less tentative) decision that measures of the weaker spectrum are not reliable is a common reason for (more or less tentatively) rejecting the system as one that will not provide reliable absolute dimensions. The most serious difficulties are (a) marginal or sub-marginal appearance of the lines of the weaker component and (b) blending. The latter effect is particularly serious for the broad hydrogen lines in B- and A-type spectra, and for the many weak lines in later types. I have preferred to adopt the conservative approach, and to disqualify systems tentatively in doubtful cases.

For systems in which the fainter component is probably of spectral type F or later, it may be possible to rescue or improve the reliability of radial-velocity measurements by going to the visual region. This approach is typified by the work on Z Her (Popper, 1956), and is being pursued for a number of systems at Mount Wilson. The visual region offers several advantages over the photographic for systems in which the fainter component is also the redder and is of advanced type.

It should be apparent that in those cases where the radial-velocity measurements of the weaker component are unreliable for the reason discussed, the assigned spectral types are not likely to be of much weight except, of course, in cases of total eclipses.

(2) The second problem concerns use of the *B,V* colour system for photometry. It is essential that eclipsing binaries be observed in the *B,V* system (or better, the *U,B,V* system, three colours being especially desirable for early-type stars), or some system equally well calibrated against spectral type. Aside from some work by Eggen, photoelectric observers have tended to observe either in a single colour or on a poorly calibrated colour system. The importance of this matter lies primarily in the following: because of the difficulty of spectral classification, particularly of the fainter components of eclipsing binaries, one is unable to tell reliably what kind of a star one is concerned with unless the colour is known. An illustrative example may serve to emphasize the point. Both components of Z Her had been classified as F-type stars. From the depths of minima, Plaut (1950) concluded the secondary was a G4 dwarf. It is, as the colours and radii show unambiguously (Popper, 1956), a K0 subgiant. Similar discrepancies are removed by two-colour observations of Z Vul and CM Lac (unpublished), and may be expected to be for other systems as well.

Other important reasons for using the *U,B,V* colour system are (a) in computing luminosities from radii and surface temperatures, the latter are best evaluated from precise colour measurements in a standard

system; (b) the consistency check of colour against surface brightness ratio, mentioned earlier, is of value only for a completely calibrated colour system; (c) for space-reddened O and B stars, the unreddened colours can be obtained by the method of Johnson and Morgan (1953).

Although for a thorough analysis it may be desirable to obtain complete light curves in two or three colours, in many cases more limited two-colour observations will suffice.

Application of some of the principles discussed in sections A and B may be found in three of the writer's papers (1955, 1956a, 1956b).

C. Classification of Systems

Using Kopal's list as a basis, I have prepared a tentative classification of each system into one of the following five groups.

(a) Reliable systems.

(b) Systems which should be reliable when more data are obtained.

(c) Border-line cases. A more thorough analysis of existing observational data than I have attempted, or improved observational data are required to decide whether such a system can give reliable results.

(d) Systems with insufficient information to tell whether to classify under (a), (b), or (c).

(e) Systems probably not capable of providing reliable absolute dimensions. These systems are not discussed further or listed.

We understand by a "reliable system" one that can provide reliable values of masses, radii, and colours of the two components. In many of the cases classed as unreliable, either the spectrographic or the photometric results may be highly determinate, but not both.

For the sake of convenience, the eclipsing binaries are divided into five groups according to the types of the stars, as follows.

A. Two similar main-sequence stars, B8 or later. W UMa stars are excluded.

B. Two O-B stars. Such famous systems as u Her and μ^1 Sco are omitted because the light curves, by their nature, are not conducive to reliable analysis.

C. Dissimilar stars, both of which are on or near the main sequence.

D. Systems with one or two subgiant components. For this purpose, we define a subgiant as a star with a spectral type not far from K0 and a radius in the range of 2 to 5 solar radii.

E. One or both components of high luminosity, luminosity class III or higher. Because of the very small number of such systems, and the difficulties in their analysis, the criteria for reliability in these cases should be relaxed.

In the following tables are tentatively grouped those stars of Kopal's list (1955) considered likely to provide reliable basic data. A few systems not given by Kopal have been added. The tables give, for each star classified according to reliabiilty, its name and spectral type, plus an indication of what type of observational work is needed, either spectrographic (Sp) or photometric (Pm), in order to consolidate knowledge of the system. In the "Need Sp" and "Need Pm" columns, an exclamation point (!) indicates the observations are badly needed; an asterisk (*) that more are desirable; the letter "d" or "(d)" that I have unpublished observational data of a more "d", or less, "(d)", definitive nature. These last symbols are *not* intended to establish priority or sole claim to observation of any system. More duplication of observations than has been the custom in the past would be advantageous.

Finally no attempt is made here to list actual numerical results. Kopal gives such results, in addition to references to the original observations.

A. Similar Main-Sequence Stars

Star	Type	Need Sp	Need Pm	Star	Type	Need Sp	Need Pm
	Class a				Class b		
AR Aur	A0		*	RX Her	A0	d	
β Aur	A0			ZZ Boo	F0		!
WW Aur	A7		*	VZ Hya	F5	*	!
YY Gem	M1			WZ Oph	G0	*	d
				UV Leo	G0	!	

B. Two O-B Stars

Star	Type	Need Sp	Need Pm	Star	Type	Need Sp	Need Pm
	Class a				Class c		
Y Cyg	B0			V382 Cyg	O9		!
U Oph	B5	*		AH Cep	B0.5		*
	Class b			V Pup	B2	*	*
V478 Cyg	B0		!	AG Per	B3		*
CW Cep	B3		d		Class d		
TT Aur	B3, B7		!	CC Cas	O9	*	!
DI Her	B5	*	!	V453 Cyg	B1	*	!

C. Dissimilar Main-Sequence (?) Stars

Star	Type		Need Sp	Need Pm
	Class d			
CM Lac	A2	F0	(d)	d
V477 Cyg	A3	F5	*	(d)
TX Her	A5	F0	*	(d)
RT And	F8	K1	!	

D. Systems With One or Two Subgiants

Star	Hotter Type	Need Sp	Need Pm	Star	Hotter Type	Need Sp	Need Pm
	Class a				Class c		
Z Her	F4			U Cep	B8	!	
AR Lac	G2			U Sge	B8	*	
	Class b			TW Dra	A5	!	*
RS CVn	F4	d	(d)		Class d		
RZ Eri	F5	(d)	!	RW UMa	F9	!	!
SS Cam	F5	!	!	VV Mon	G0	!	!
WW Dra	G2	!	!	AW Her	G2	!	!
RT Lac	G8	*	!				

E. High Luminosity Stars

Star	Types	Need Sp	Need Pm	Star	Types	Need Sp	Need Pm
	Class a				Class c		
V356 Sgr	A2 II, B3	*	*	TW Cnc	K0 III, A7	d	(d)
Z Vul	A5 III, B5	d	d	KU Cyg	F4 Ia?, M0 II	(d)	(d)
	Class b			ζ Aur	K4 Ib–II, B7	d	*
SZ Psc	K1 III, A	!	!	31 Cyg	K2 Ib, B5	!	!
RZ Cnc	K2 III, K4 III	d	(d)		Class d		
				AL Vel	K0 III, A3	!	!
				TU Mon	A5 III?, B5	!	!
				UX Mon	A5 III?, G0 III?	!	!
				V777 Sgr	K5 Ib, A	!	!

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Comments made by A. P. Linnell:

Dr. Popper's plea that photoelectric observations be obtained in at least two colours deserves strong emphasis. It is important to point out that if observers standardize on a particular system, say the *U,B,V* system, it will still be necessary to perform extensive reductions in combining results of different observers, because of the variations in colour sensitivity from one phototube to another. Observers using refractors may be handicapped by the ultra-violet absorption of the telescope objective.

Comments made by L. Plaut:

Dr. Popper proposed to provide a list of elements of reliable systems. This

list would show a selection of systems of a certain kind. I would prefer to give a list of systems complete down to a certain magnitude or, better, in a certain volume of space. All elements have to be provided with their mean errors (or estimates of them) showing the amount of their reliability. Such a list would be of more interest to those engaged in statistical and evolutionary problems.

THE MASSES OF THE O- AND B-TYPE SPECTROSCOPIC BINARIES, DETERMINED BY THE MASS-LUMINOSITY RELATION

By Joseph A. Pearce

In spite of the large number of photometric and spectroscopic orbits of binary systems which have been computed, our knowledge of the masses of the stars is comparatively meagre, being derived solely from those few systems for which accurate spectroscopic and photometric elements exist. Recently, owing mainly to the spectrophotometric techniques developed by R. M. Petrie at Victoria, a hopeful advance has been made by the accurate determination of the magnitude differences and luminosities of the components of spectroscopic binaries. These data, in conjunction with the mass-luminosity relation, enable one to estimate the orbital inclinations, and hence the masses may be deduced within reasonable uncertainties.

Making use of Kuiper's mass-luminosity relation, the author successfully predicted that certain spectroscopic binaries such as H.D. 34333, 44701, 227696, would prove to be eclipsing systems, and subsequently eclipses predicted for other binaries investigated at Victoria, such as H.D. 193611 by Miss J. K. McDonald and H.D. 218066 by R. M. Petrie, were verified by S. Gaposchkin. For many eclipsing systems having $i < 70°$, the interpretation of the light curve is completely ambiguous without the aid of the spectrophotometric data relative to the light of the secondary star.

It may be of interest to the members of this Binary Star Conference to discuss the method employed by double-star investigators in recent years to evaluate the masses of those O- and B-type systems for which the requisite data, both spectroscopic and photometric, are most accurately known. The systems selected include many of the well-known variables, and also a number of massive systems whose orbits are unpublished, recently investigated by the author.

It is tacitly assumed that all systems may be represented by the same mass-luminosity relation, $\log \text{mass} = +0.548 = 0.144\ M_c + 0.002\ M_c^2$, which was derived by Petrie solely from spectroscopic binaries, no eclipsing systems being used for this comparison.

The computed inclinations depend critically upon the accuracy of the spectroscopic data, the adopted temperature scale, the bolometric corrections, and the slope of the mass-luminosity relation, for it is evident that if we assign too high a bolometric magnitude to the components, the

resulting orbital inclination of the system will be too low, and in consequence the deduced masses will be too large.

The adopted temperatures and bolometric corrections for these systems are given in Table I. For the O5-B0 stars, Petrie's ionization temperatures are preferred to Kuiper's temperatures since the maximum of He II is certainly at O5 (or even earlier), as adopted by Petrie. If Kuiper's higher temperatures are used for these stars the computed orbital inclinations of the O-type systems are very definitely too low, as compared with the photometric determinations. For the B1-A0 stars, Kuiper's temperature scale is adopted, as also are his bolometric corrections.

TABLE I

ADOPTED TEMPERATURES AND BOLOMETRIC CORRECTIONS FOR THE O- AND B-TYPE STARS

Spectral Type	Temperature T_e	$M_b - M_{pv}$	$M_b - M_{pv} + 2 \log \frac{T_e}{5200°}$
O5	36300° K.	−3.60	−1.91
O6	34600	−3.48	−1.83
O7	32900	−3.36	−1.76
O8	31700	−3.27	−1.70
O9	30700	−3.18	−1.64
B0	28600	−3.00	−1.52
B1	22900	−2.48	−1.19
B2	20400	−2.22	−1.03
B3	18600	−2.01	−0.90
B4	17000	−1.79	−0.76
B5	15500	−1.58	−0.63
B6	14500	−1.42	−0.53
B7	13400	−1.23	−0.41
B8	12300	−1.04	−0.29
B9	11500	−0.88	−0.19
A0	10700	−0.72	−0.09

The pertinent data for each system are displayed in Table II. The spectral types and minimum masses come directly from the spectroscopic observations; the measured spectroscopic luminosities M_{pv} for each component are largely due to Petrie; the photometric inclinations i_{ph} are taken from Plaut's recent catalogue supplemented by recent values for the four systems nos. 1, 14, 29 and 34.

The spectroscopic inclinations i_{sp} are computed from Petrie's mass-luminosity relation. The masses in the final columns of Table II were computed from the observed minimum masses and the photometric inclinations, in the usual manner.

The positions of each system on the M-L diagram are shown in the figure, the primary components being plotted as solid circles and the secondary components as open circles. For many systems such as H.D. 215835, Y Cyg, CC Cas, AH Cep and H.D. 37756, the values of i com-

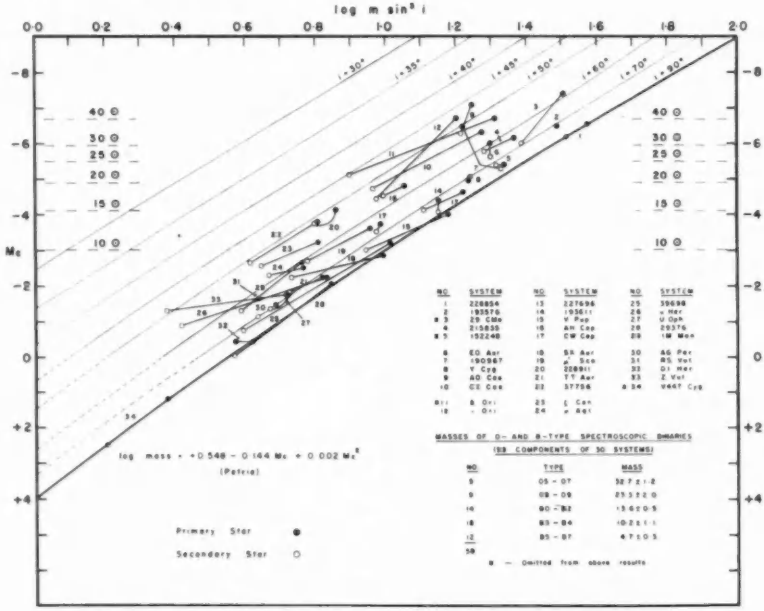
TABLE II

MASS AND LUMINOSITY DATA FOR O- and B-TYPE BINARIES

No.	System	Spectral Types		log $m \sin^3 i$		M_{pv}		Inclination		Δi	Masses	
		A	B	A	B	A	B	i_{ph}	i_{sp}	$(ph-sp)$	A	B
1	228854	O6.5	O7.5	1.572	1.514	(−4.73	−4.45)	89°.3	(90°)	(0°)	37.3⊙	32.7⊙
2	193576	O5.5	WN5.5	1.489		−4.6		73.5	70	+4	35.0	
3	29 CMa	O7	B1	1.508	1.386			64			(44.3	33.5)*
4	215835	O5.0	O5.5	1.369	1.281	−4.2	−3.9		64		32.2	26.3
5	152248	O9	O9	1.340	1.318	−3.8	−3.8		75		(24.3	23.1)*
6	EO Aur	B3	B3	1.296	1.295	−5.1	−4.6	62.4	60	+2	28.4	28.3
7	190967	B1	O9.5	1.220	1.328	−5.2	−3.6	80.3	80	0	17.3	22.2
8	Y Cyg	O9	O9	1.236	1.241	−3.3	−3.4	88.2	71	+17	17.2	17.4
9	AO Cas	O8.5	O8.5	1.246	1.215	−5.4	−4.6	57	47	+10	29.8	27.8
10	CC Cas	O8	O8	1.276	0.964	−4.6	−3.0	76	55	+21	20.7	10.1
11	δ Ori	O9.5	B0	1.312	0.897	−5.1	−3.6	67	53	+14	(26.3	10.1)*
12	ι Ori	O8.5	O8.5	1.201	0.971	−5.04	−2.76		47		40.6	23.9
13	227696	B1.5	B1.5	1.222	1.107	−3.5	−3.0	78	77	+1	17.8	13.7
14	193611	B0.5	B0.5	1.152	1.158	−3.0	−2.7	82.2	73	+9	14.6	14.8
15	V Pup	B1	B3	1.180	0.944	−2.8	−2.1	76.7	90	−13	16.4	9.5
16	AH Cep	B1	B1	1.057	0.991	−3.6	−3.3	66.1	57	+9	14.9	12.8
17	CW Cep	B3	B3	0.988	0.978	−2.8	−2.6	81.9	66	+16	10.0	9.8
18	SX Aur	B3.5	B6	0.994	0.722	−2.2	−1.3	79.2	84	−5	10.4	5.6
19	μ′ Sco	B3	B7	0.959	0.778	−2.7	−2.3	60	65	−5	14.0	9.2
20	228911	B2	B2	0.857	0.799	−3.1	−2.7		52		14.7	12.9
21	TT Aur	B3	B3	0.826	0.724	−1.2	−0.9	85.8	75	+11	6.8	5.4
22	37756	B1	B1	0.803	0.611	−2.6	−1.5		52		13.0	9.3
23	ζ Cen	B1.5	B2.5	0.806	0.643	−2.1	−1.6		57		10.8	7.5
24	σ Aql	B3	B3	0.766	0.664	−1.6	−1.4	70.7	63	+8	6.9	5.5
25	39698	B3	B5	1.013	0.591	−2.3	−0.1		81		10.7	4.0
26	u Her	B3	B7	0.832	0.415	−1.3	−0.5	76.2	75	+1	7.4	2.8
27	U Oph	B5	B5	0.722	0.666	−1.1	−0.7	86.9	70	+17	5.3	4.7
28	29376	B4	B5	0.845	0.568	−1.3	+0.6		84		7.1	3.8
29	IM Mon	B4	B7	0.762	0.584	−1.9	−0.9	62	60	+2	8.4	5.6
30	AG Per	B4	B5	0.687	0.632	−0.7	−0.5	8: 1	71	−10	5.1	4.5
31	RS Vul	B5		0.637		−1.0		80	63	+17	4.5	
32	DI Her	B5	B4	0.570	0.616	+0.2	+0.3	89	90	−1	3.7	4.1
33	Z Vul	B4	B6	0.719	0.373	−1.0	−0.8	88.9	69	+20	5.2	2.4
34	V447 Cyg	A3	F5	0.374	0.204	(+1.1	+2.2)	88.5	(90)	(−2)	(2.4	1.6)

* These systems were omitted from the discussion. The luminosities for systems Nos. 1 and 34 were not spectrophotometrically determi ned but were adopted from the photometric solutions. They are omitted in the Δi comparison.

puted from the secondary components are in excellent agreement with those computed from the primary stars, since the slope $q = \Delta \log \text{mass}/\Delta m$ is parallel to the *M-L* curve. For a number of systems, however, it is quite evident that the secondary stars are over-luminous, being too bright for their masses.



For the 23 eclipsing systems (excluding nos. 1, 4 and 34) we find:

Δi (ph-sp) $= +7° \pm 1.°2$, from the primaries,

Δi (ph-sp) $= +13° \pm 1.°6$, from the secondaries.

In the brief discussion which follows, the secondary components are disregarded since for a number of systems in which Δm is large, the secondary spectra are relatively faint and in consequence the luminosities and masses are of lower accuracy.

It is not immediately clear why these well-determined spectroscopic luminosities in conjunction with Petrie's mass-luminosity relation (derived solely from spectroscopic binaries) should give orbital inclinations consistently smaller than those deduced from the light curves. The systematic error may be associated with the observed luminosities, or the adopted temperature scale, or the slope of the mass-luminosity relation, or the photometric inclinations. The difference of 7° in the orbital inclination would vanish if we were to consider that the Victoria spectro-

scopic absolute magnitude system is affected by a systematic error of 0.4 magnitudes, i.e. the observed luminosities are too high, and the stars too bright. Many lines of evidence preclude the possibility of an error of this magnitude. On the other hand, the inclinations could be harmonized by decreasing the temperature scale some 6000° for the O stars and 3000° for the B stars but this is an equally improbable solution.

The photometric inclinations for these well-observed eclipsing systems cannot be systematically too high, and we are left with one possibility only of removing the discrepancy, namely by a change in the slope of Petrie's *M-L* curve. The zero point of the relation: log mass $= 0.548 - 0.144\ M_c + 0.002\ M_c^2$ was fixed upon Sirius, but the slope was determined by the measured Δm and mass-ratios of ninety-three spectroscopic binary systems.

Accepting the observed luminosities, bolometric corrections and photometric inclinations, as given in Table II, a solution from the 23 eclipsing variables gave a slope of $q = -0.126 \pm 0.0010$. This, in effect, raised the *M-L* curve from Petrie's position to those of Kuiper's and Eddington's curves which are approximately half a magnitude higher in the region under consideration i.e. M_c from 0 to −5. Admittedly the stars in Table II are too few upon which to base a revision of the slope of the *M-L* relation, but the conclusion from the data herein given is inescapable: this method of deriving orbital inclinations of eclipsing or non-eclipsing binaries is valid only when the *M-L* relation which satisfies the eclipsing variables is used in conjunction with accurately measured luminosities.

For the present it seems desirable to compute the masses of the O- and B-type binaries by adopting the photometric inclinations, in the usual manner. Excluding 10 components the masses of 58 O- and B-type components of 30 systems were found to be:

No. of Components	Spectral Types	Masses
5	O5–O7	$32.7 \pm 1.2\ \odot$
9	O8–O9	23.3 ± 2.0
14	B0–B2	13.6 ± 0.5
18	B3–B4	10.2 ± 1.1
12	B5–B7	4.7 ± 0.3

Comments made by D. M. Popper:

Dr. Pearce is the acknowledged master of radial-velocity measures of early-type spectroscopic binaries. A striking tribute to his skill is the naming of H.D. 215835 as a variable star (DH Cep) on the basis of Dr. Pearce's prediction from his velocity measures that it *should* be an eclipsing variable.

In Dr. Pearce's paper, in mine, and in two or three others, reference has been made to Dr. Petrie's basic work in determining the magnitude differences of the components of spectroscopic binaries. I raise the question of the

reliability of the Δm results in cases where the two stars are not quite similar in luminosity, surface brightness, and even mass. The fundamental assumption made in the Δm determinations is that the lines of the two stars of the system have the same depth or equivalent width (depending on whether depths or equivalent widths are measured), or that the differences in depth or equivalent width can be evaluated from the difference in spectral type determined from the measures of the lines themselves. I question the validity of this fundamental assumption unless the two stars are quite similar. In general, the larger the difference in magnitude and in spectral type, the less reliable the Δm determination will be. A complicating factor, in addition to the dependence of line profiles on luminosity and spectral type, is that stars of appreciably different mass and radius may well have different rotational velocity.

That spectral type difference may not always be determined precisely from the line measurements is shown by some examples. Petrie gives F2 and F5 for Z Her, A2 and A2 for CM Lac, and B4 and B6 for Z Vul, whereas the spectral types of the secondary components are more nearly K0, F0, and A5 respectively.

Two additional causes of inaccuracy in the Δm results are the uncertainty of the positions of the lines of the secondary where Δm is large and the systematic dependence of the equivalent widths of the hydrogen lines on absolute magnitude.

Dr. Petrie pointed out that the former of these two effects is kept very small by the methods used, and that correction for the latter effect has been incorporated in the results of his more recent papers.

Comments made by ANNE B. UNDERHILL:

The present paper and that by D. M. Popper have illustrated how important the bolometric correction is in estimating the bolometric absolute magnitude of the early-type stars. I should like to draw your attention to the results of model-atmosphere calculations of the bolometric corrections and of the effective temperatures of O and B stars. These results differ somewhat from the values proposed in the papers we have heard, the computed bolometric corrections and effective temperatures being generally larger than the values quoted. I should like to remind you of the physical difference between effective temperature, brightness temperature, colour temperature, and excitation or ionization temperature. In the O and B stars these temperatures may differ by several thousands of degrees. A brief summary of computed results follows. The spectral types are estimates from the general properties of the models and of the computed spectra. For the unpublished models the spectral types are rough estimates which will be improved later.

Spectral Type	Log g	Boundary Temp.	Effective Temp.	Bolometric Correction (mag.)	Author	Reference
O5	4.20	35,000°	44,600°	−4.31	Underhill	*Ap.J.*,115,577,1952
O9	4.20	25,200	36,800	−3.79*	Underhill	*Ap.J.*,115,577,1952
B1 V	4.48	17,700	27,300	−3.00	Pecker	*Ann.d'Ap.*,13,433,1950
B1.5 V	4.48	17,300	26,360	−2.83	Pecker	*Ann.d'Ap.*,13,433,1950
B1	3.80	18,800	28,470	−2.91	Underhill	Unpublished
B1	4.20	18,800	29,500	−3.04	Underhill	Unpublished
B2	3.80	16,800	22,700	−2.63	McDonald	*Ap.J.*,115,577,1952
B3	3.80	16,800	27,000	−2.85	Underhill	Unpublished
B3	4.20	16,800	27,870	−2.94	Underhill	Unpublished

*Corrected value.

PROBLEMS OF CO-ORDINATION AND CO-OPERATION IN DOUBLE-STAR ASTRONOMY

By Frank Bradshaw Wood

One of the most difficult and most profitable tasks in modern astronomy is the combination of different sets of observational evidence in an attempt to get more information than is possible by merely summing up that which can be deduced from each single set. A simple example is that of an eclipsing variable for which photometric and spectrographic orbits are available. Neither set alone can give absolute dimensions, but the combination of the value of the inclination found photometrically with the $a \sin i$ in kilometres found from the velocity curve gives the semi-major axis of the orbit in absolute units. From the light curve are found the radii of the stars in terms of the size of the orbit; thus, the sizes of the stars in absolute units are computed. A similar treatment can be applied to determine masses.

This is perhaps the simplest case in common use; one does not combine the observations themselves, but rather combines the deductions from each set and gains knowledge in excess of that yielded by merely writing down the results of both solutions. Yet even in this simple case, complications can arise. For example, what does one do when different values of eccentricity are found by the two methods? No inviolable rule or computational method can be laid down. The decision must be a matter of judgment and of weighing the strength of the various lines of evidence—a process which must take into account the accuracy of the observations, the nature of the system, and also almost inevitably the fact that the observations were made at different epochs. To do this properly, an astronomer should have had experience in photometry and in spectrography both in observing and in making the reductions and orbital computations. Yet such is the degree of specialization required, that only a few people in double-star astronomy are able to contribute to both fields. One service which this conference might perform is to emphasize the desirability of a spectrographist doing a little photometry, or a photometric specialist obtaining a few velocity curves. It might even be possible to work out an exchange arrangement between institutions which specialize in each field.

Of course, we must realize that the above desideratum may frequently be difficult to obtain. Yet the problem cannot be avoided, since an observer trying to deduce from his observations all possible knowledge of the physical nature of the system under consideration cannot do so

without taking into account all the observed evidence. The next best solution is an effective means of communication between individual workers and a mutual appreciation of both the strengths and the inherent weaknesses of the observational evidence of both types. It is very difficult, when attempting to draw a consistent picture from apparently conflicting evidence, to refrain from one of two extremes. One is to fail to consider seriously evidence from other sources which seems to conflict with that which the observer himself has found. The other—found less often because of reasons of human psychology—is that in which the observer is so aware of the possible sources of error in his own observations that he gives evidence from other sources higher creditability than it merits. Informal exchange of information and ideas, well in advance of publication, seems to offer much promise in the search for a middle ground. We can hope that this conference will be an important factor in emphasizing the desirability of this and in discussing means of more effectively achieving it.

Part of the story of U Cephei may illustrate this need. R. S. Dugan observed the light curve and, while some unexplained peculiarities were present, he found clearly that the orbit was circular or nearly so. He had used the polarizing photometer—an instrument extraordinarily free from systematic error—and there was no question of the central location of secondary minimum or of the fact that its duration was approximately that of primary. Some time later, E. F. Carpenter obtained a velocity curve which clearly indicated, on any conventional treatment, an orbit of considerable eccentricity. The discrepancy was real. It could not be rationalized on the basis of existing data. Shortly after completing the velocity curve, Carpenter went to the Steward Observatory where no spectrograph was available; twenty years passed before anyone thought seriously enough of the discrepancy to obtain another velocity curve. Then Otto Struve confirmed the fact that normal treatment of the radial-velocity observations gives a distorted curve and further found that the amount of distortion was not constant. Finally, the work of Robert Hardie suggested the physical cause for the apparent discrepancy—some twenty-five years after it was evident that the problem existed.

It would be easy to continue with other examples, but I think the conclusions are reasonably clear. The first, of course, is that it is always desirable to have the different types of observations made as nearly simultaneously as possible. The next is, that when discrepancies exist, the observations be discussed with a clear awareness of the types of errors which may have entered in observation, reduction, and computation, in order to be certain that the apparent discrepancy is a real one. When the

necessary experience cannot be combined in one person, co-operation between two or more will be necessary. Finally, of course, if real discrepancies exist, it should be realized that these offer an opportunity, either by theory or by continued observation or by both, to improve our present incomplete knowledge of double stars. It is, for example, from this sort of awareness that has risen the present concept of streams or shells of gas associated with these systems.

Of course, this is by no means limited to explaining discrepancies. For example, in the case of a star like ζ Aurigae it is of interest to try to add the total line absorption in a given spectral region at a given phase and compare this with the photometrically observed loss of light in the same region. The results are interesting and their implications go well beyond that which may be deduced from either source alone.

So far, I have not mentioned the third field represented here—visual binaries. Probably because these of necessity deal with pairs greatly more separated than normal eclipsing systems, co-operative effort has been rare. Yet, for this reason, it is probably all the more important that it be sought in all possible cases. This is especially true since we know that many eclipsing systems are notorious violators of any conventional mass-luminosity relation and thus that outside aid must be sought whenever possible. One effort to see what can be obtained from a combined photometric-astrometric attack is now under way at the Flower and Cook Observatory. Lawrence Frederick is observing the systems VV Cephei and ϵ Aurigae photoelectrically during their current eclipses. He also has the results of thirty years of astrometric observations at Sproul Observatory, and he will attempt to see what sort of information can be gained by the combination.

Thus far, I have discussed only the problems which arise when we attempt to combine the results of different kinds of observations but not the observations themselves. Quite a different type of problem occurs when we try to combine photometric observations made with different instruments which inevitably have somewhat different colour responses. In any normal case, I believe most photometrists would agree that this is a dangerous procedure and one which should be avoided. That is, simply to combine observations made with different instruments and to compute a solution from these is a risky practice indeed and is likely to yield results devoid of real physical significance. As Dugan once remarked in a somewhat different connection, this yields a definite answer, but the answer is usually wrong. Yet there are cases—ϵ Aurigae and VV Cephei are the ones of greatest current interest—where we must of necessity combine various sets of observations if we are to get any satisfactory

picture. When eclipsing stars have periods of from two to thirty years, and when they have in addition shown signs of intrinsic fluctuation, we have little choice except to observe as intensively as possible at each eclipse and then to combine the observations to try to get a continuous record of the changes. It is in the photometric observations that the difficulties are the greatest, and this is especially true when the system shows a large change of colour during the eclipse.

An example of the difficulty of this sort of combination is shown by the discussions of the 1939–40 observations of ζ Aurigae. Three different interpretations of the existing observations concluded (1) that the eclipse was by a sharply opaque limb of the larger star, (2) that the observations could be interpreted as such a conventional eclipse, but that they were represented better by the concept of an atmospheric eclipse, and (3) that their interpretation required the postulate of a large cloud in the chromosphere. Not until the multi-colour observations of 1948–49 was it possible to show clearly that the eclipse was primarily an extinction phenomenon. One of the chief difficulties in the past has been in the use by different observers of comparison stars of different colour. This was enhanced by the relatively wide colour bands which had to be used before the development of satisfactory interference filters. Another has been the lack of communication, so that an observer sometimes had to wait years after an eclipse in order to learn whether his observations were of value in a general picture.

Under the leadership of D. J. K. O'Connell, President, and J. E. Merrill, Secretary, Commission 42 of the International Astronomical Union is attempting to eliminate some of these difficulties in the current eclipses of ϵ Aurigae and VV Cephei by disseminating in advance as much information as is available—the names of astronomers and observatories at which the stars are being observed, types of observation, wave-length ranges, comparison stars used by photometric observers, and abstracts of results. On the instructions of Dr. O'Connell, Dr. Merrill sent a circular letter to all members of commission 42, to all observers who, according to our records, had published papers on one of these stars in the past, and to a few others strategically located in longitude. I have served as co-ordinator, and to date three bulletins have been circulated to all respondents. The replies have been heartening—especially so, since in at least three places, astronomers who had not planned observations, decided to do so as part of a co-operative effort. (A quite independent parallel development has been a similar appeal from C. de Jager at Utrecht for observation of the β Canis Majoris star 12 Lacertae in an attempt to follow it without interruption throughout two beat periods.)

To date, astronomers at twenty different observatories have indicated their intention of observing these systems. In several cases, observations have been under way for some time; in others, participation was in direct response to the circular letter. Information concerning such matters as comparison stars and wave-length regions being observed have been circulated to the thirty astronomers who indicated their intentions of participating.

As might be expected, most of the participating institutions are in North America or western Europe. The large Pacific-Asian longitude gap is filled photometrically only by Tokyo Observatory; spectrographically, the easternmost location is the Crimean Astrophysical Observatory and the westernmost, Victoria.

At present, the participants write either to Dr. Merrill or to me, and at suitable intervals, a bulletin summarizing the information is distributed to all participants. It may well be that this same procedure will be satisfactory in disseminating the observational results. In any case, the experience which will undoubtedly be gained by such attempts should be valuable in future efforts. Certainly there are many fields of variable star photometry in which continuous observation over a specified interval could yield valuable results which can be obtained no other way.

THE ANALYSIS OF THE CHROMOSPHERE OF ZETA AURIGAE

By O. C. Wilson

Some Observations

It is very difficult to obtain information about the outer atmospheres, i.e. the chromospheric regions, of stars. Unlike the sun, whose surface phenomena can be explored in considerable detail, the stars must always be observed in integrated light. In the spectra of normal stars of later type, probably the only features which can be identified as definitely chromospheric in origin are the emission components often seen at the centres of H and K of Ca II. Shell stars are easily recognizable by the appearance of their spectra and some late-type stars, e.g. α Her (Deutsch, 1956), possess extended envelopes which give clear spectral evidence of their presence. These shells and envelopes, however, are scarcely to be classed as normal chromospheres.

These remarks illustrate the importance of eclipsing binaries such as ζ Aurigae. In these systems both stars are of comparable brightness, but one is large and cool, the other small and hot. Whenever the small star passes behind the atmosphere of the large one, toward or away from totality, it serves as an approximately continuous source of radiation and the atmospheric constituents of the large star produce absorption lines in its spectrum. Measures of intensity and position of these chromospheric absorption lines, extended as far outward from totality as possible, provide the raw material for a study of the circumstances prevailing in the chromosphere of the large component.

One must ask whether the data obtained in this fashion apply to a "normal" chromosphere. In the case of ζ Aurigae the answer is neither yes, nor an unqualified no. During totality the spectrum of this star shows no abnormalities; there is nothing to suggest, for example, that it has a shell or envelope of any kind, and the appearance of the emission components of H and K are quite normal for its luminosity. On the other hand, subsequent discussion of the circumstances shows that the radiation of the blue companion is largely responsible for the level of ionization observed in the chromosphere of the cool star during the time of chromospheric observation. Whether the influence of the hot star goes beyond this is not known.

Spectrographic observation is largely restricted to wave-lengths below λ4000 since interference from the underlying K-type spectrum is thus

minimized. Lines selected for intensity measurement are those whose f-values are known, or can be deduced with reasonable accuracy, since these lines are the ones useful in forming curves of growth for the various chromospheric levels. In the most recent work (Wilson and Abt, 1954) curves of growth were constructed for the atoms and for the ions separately, as well as for the lines of the Balmer series of hydrogen.

Once the curves of growth have been completed, certain physical information may be readily extracted from them. For the 1939–40 and 1947–48 eclipses of ζ Aurigae the results are summarized herewith:

1. *Turbulence.* By fitting the observed curves of growth to theoretically calculated curves, the quantity $\Delta\lambda_D$ may be found for each line of sight through the chromosphere. It turns out that $\Delta\lambda_D$ for the ions is slightly, though consistently, larger than for the atoms. Corresponding turbulent velocities are about 8 km./sec. for the atoms, constant over the observed height range, and from 10 km./sec. for the ions in the lower levels to 13 km./sec. in the upper. $\Delta\lambda_D$ for hydrogen is consistent with these values.

2. *Excitation Temperatures.* These have been found from two excited states of Fe I and one of Ti II. Although the results scatter a good deal, there is no doubt that T_{ex} rises from about 3500° in the lowest observed chromospheric levels to about 7000° in the highest. The increase is linear as far as one can tell from the plotted values.

3. *Apparent Gradients.* Values of log N for the various atoms and ions may be read from the curves of growth. N is the number of particles along the line of sight responsible for the production of a particular line. Since the excitation temperature is a function of height, part of the change in N between successive atmospheric levels for lines arising from excited states will be due to the increased relative population of the state at the higher temperature of the upper level. Such values of log N were therefore corrected to the ground states by application of the Boltzmann factors corresponding to the excitation temperatures deduced previously. When this is done, and the various log N values are plotted against height, the resulting graphs have been called apparent gradient curves. It turns out that these curves have virtually the same shape for nearly all atoms and ions, all being steepest at the lesser heights and then tending toward approximate linearity further out. The curves for the neutral particles terminate at a height of about 28×10^6 km. because it is about here that their lines begin to "slide" rapidly down the linear part of the curve of growth and quickly disappear.

4. *H, Ca I, and Ca II.* The apparent gradient curve for the first excited state of hydrogen is different. It is very flat out to about 25×10^6 km. but then bends over and has about the same slope in the higher levels as do

the gradient curves for the metals. Curves for Ca I (λ4227) and Ca II (K) are also different, though not in the same way as for hydrogen. The Ca I curve is much steeper than those of the other metals; in fact it is much the steepest of all. For Ca II the curve is somewhat steeper than the other metals at the lowest chromospheric levels, but is about the same further out.

5. *Differences between Various Transits.* The apparent gradient curves for all particles agree in one respect: The curves for ingress and egress of 1939–40 are very similar to those of egress 1947–48. But all the curves for ingress 1947–48 deviate, in the higher levels, in the direction of a slower gradient and correspondingly larger log N values at given heights.

6. *Degree of Chromospheric Ionization.* On a plate taken December 18, 1939, at a height of 14.3×10^6 km. during ingress, all the chromospheric lines between 3070 and 3970 A. were measured and nearly all identified. For the metals the observed numbers of lines of the neutral and singly ionized stages were as follows:

Numbers of Observed Lines, $H = 14.3 \times 10^6$ km.

Element	I	II	I.P.	Element	I	II	I.P.
Na	1	0	5.12	Cr	4	36	6.74
Mg	4	0	7.61	Mn	0	9	7.40
Al	2	0	5.96	Fe	62	28	7.86
Ca	0	2	6.09	Ni	9	2	7.61
Sc	0	18	6.7	Y	0	6	6.5
Ti	0	114	6.81	Zr	0	9	6.92
V	0	22	6.71	Yb	0	1	6.23

For Na, Mg, Al, Ca, Mn and Yb the results are those to be expected from the nature of their spectra. But Sc, Ti, V, Cr, and Y have strong lines of both neutral and singly ionized stages within the wave-length region considered. Ni II has no outstanding lines in this region and the Fe II lines available do not arise from the ground state nor are they very intense in the laboratory. One must conclude that both Ni and Fe are rather heavily ionized and that the ionization of the metals with I.P. about one volt less is virtually complete. Also it is clear from the table that only iron yields enough lines of both neutral and singly ionized stages to be of much use in discussing the ionization in detail, but unfortunately the f-values for the Fe II lines are unknown. Apart from iron, only Ca II and Ca I (λ4227) are available.

7. *Radial Velocities from Chromospheric Lines.* The velocities from the chromospheric lines were measured in detail on many plates. It is impossible to discuss them here except to mention the following results: (a) Among the metallic lines careful search revealed no velocity differ-

ences between groups of strong and weak lines, nor between lines of ions and of atoms. (b) Hydrogen velocities usually agreed closely with those of the metals. (c) Ca II velocities sometimes deviated appreciably from the others but mostly in the lower levels where H and K are strong and square-sided.

Some Deductions

1. *Spectroscopic and Photometric Gradients.* Roach and Wood (1952) derive from the photometry of the 1947–48 eclipse that a has the value 5.1×10^{-12} cm.$^{-1}$ where a is the parameter appearing in the assumed expression for density variation: $n = n_0 e^{-ah}$. The spectroscopic gradients are not accurately determined in the lowest levels and indicate merely that $a > 4 \times 10^{-12}$ cm.$^{-1}$. To this extent they are consistent with the photometric results.

2. *Chromospheric Ionization.* From reasonable values of the temperatures and radii of the two stars, it follows that the radiative fluxes are equal in the chromosphere of the K-star for a frequency corresponding to about 4 E.V. Thus above 4 volts the ionization should be produced by the B-star radiation. The equation governing the ionization should thus be

$$\frac{n_i n_e}{n_0} = 10^{15.4-\theta I} T_* T_e^{\frac{1}{2}} W \qquad (1)$$

where T_* is the temperature of the B-star, W the geometrical dilution factor, T_e the local electron temperature, and θ is $5040/T_*$. In successively higher chromospheric levels the right-hand side of (1) should remain constant or even perhaps increase if some kind of opacity term had been included. Thus if the density of the material decreased with height as do the apparent gradient curves, one must expect a large corresponding increase in the ionization, provided that the electrons are mostly supplied by the metals themselves. However, all the gradient curves for both atoms and ions are essentially similar and there is no evidence for a change of ionization with height over a range in which the amount of material on the line of sight decreases by a factor of about 100. The evidence for this statement is very convincing for Fe; however, there remains one unexplained exception: the ionization of Ca does seem to increase upwards, and by a large amount.

If the electrons were supplied by hydrogen the same result holds, namely, the ionization of the metals should increase rapidly with height. Since, apart from Ca, the ionization of the metals is constant with height, there seems to be only one explanation: while the total amount of matter along the line of sight does decrease with height, the density of the

material does not diminish correspondingly. In fact, the density must remain essentially constant; therefore the matter must occur in condensations whose number decreases with height but whose other properties are independent of height.

3. *Condensation Densities.* Because of the fact that the only measurable ratio of ions to atoms is that for Ca, and because of the anomalous behaviour of this ratio for Ca as compared to the other metals, our knowledge of the true ionization level in the ζ Aurigae chromosphere is in a most unsatisfactory state. One can proceed only by making plausible assumptions, including the one that the solar abundance ratios prevail here also. When this is done, however, some physical specifications of the condensations may be obtained. The results are that these condensations are small (thicknesses of the order of a few hundred to a few thousand km.) and dense ($n_H = 10^{13.8}$ cm.$^{-3}$). With these specifications all of the observed phenomena, with the sole exception of the behaviour of Ca, seem to be reasonably understandable. Nevertheless, all of these conclusions should be treated with caution.

4. *Future Work.* In the light of what has been done thus far I am unable to see how extensive further observation of eclipses of ζ Aurigae can add much to our basic knowledge of how its chromosphere is constructed. The same absorption lines will simply be observed over again and unless some new approach in the method of analysis is uncovered, it is difficult to see that much progress could result. I believe it is much more important now to carry through intensive investigations of a similar nature on other suitable binaries. This will permit a variation of some of the parameters which enter into the problem and should provide a check on the validity, or lack of it, of the conclusions to which we have thus far been led.

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SOME PROBLEMS IN THE INTERPRETATION OF THE SPECTRA OF THE ZETA AURIGAE-LIKE STARS

By A. McKellar and K. O. Wright

For purposes of the present discussion we include in the group of ζ Aurigae-like stars the five systems listed in Table I. Of these systems, the best for study are ζ Aurigae and 31 Cygni. For both of these, the primary and secondary stars are not, to present knowledge, variables. The shorter period of ζ Aurigae provides the more frequent opportunities for observations near eclipse, while the longer period of 31 Cygni gives a more favourable time-scale for observations near eclipse. 32 Cygni also has non-variable component stars, but unlike the other two, has an orbital inclination further from our line of sight and so the eclipse is more nearly grazing. If, later, the value of the angle of inclination, i, can be determined for 32 Cygni, it should be a most valuable object to observe in conjunction with ζ Aurigae and 31 Cygni. All three of the above stars have K-type supergiant primaries and main-sequence B-type secondaries. VV Cephei is of much interest because it has a supergiant early M-type primary and such a long period with a correspondingly favourable time-scale for observation. However, the M-type primary is itself an irregular variable of small amplitude and has a variable spectrum, at least as regards emission lines. The secondary star is of Be type with the variable hydrogen emission associated with such stars and there is evidence of a diffuse body of gas associated with the system and having emission lines of variable intensity. These circumstances make the isolation and interpretation of the true chromospheric spectrographic eclipse effects difficult. ϵ Aurigae is the most puzzling of the five systems. Perhaps it should not even be included with the others, because, apart from the facts that a radial-velocity curve of doubtful reliability exists for the primary, that the "secondary star" makes its presence known by the occurrence of chromospheric-like lines before and after eclipse, and that an apparent eclipse does occur, there is as yet no spectrographic evidence of the secondary star.

TABLE I

Star	Spectral Types	Period	Duration of Total Eclipse	Duration of Partial Eclipse
ζ Aurigae	K+B	972 days	38 days	1.5 days
31 Cygni	K+B	~3800	61	~2
32 Cygni	K+B	~1145	13	~4
VV Cephei	M2ep+Be	~7430	~450	~22
ϵ Aurigae	F+?	~9890	~330	~192

The Mass Ratios and Dimensions of the Systems. The accuracy of determinations of the orbital and stellar dimensions of eclipsing binaries is dependent directly on the accuracy of the evaluations of the mass ratio of the components. For ordinary double-lined binaries where the stars are of similar type, the relative amplitudes of the radial-velocity curves can be determined with an accuracy of a few per cent., which is fairly satisfactory. The difficulties and uncertainties of obtaining reliable mass ratios for the components of the ζ Aurigae stars have been recognized since the first work of Christie and Wilson (1935) on ζ Aurigae. The radial-velocity curve of the late-type component star can be determined with the usual fair precision but that of the early-type component cannot. The lines of the early-type spectrum normally available for radial-velocity measurement are filled in by the late-type star, leaving only the higher members of the Balmer series of hydrogen to be seen and measured on ordinary spectrograms. These hydrogen lines, even although appearing fairly well defined on low-dispersion spectrograms are blended with the irregularly-contoured late-type spectrum. Moreover this blending is different at different phases of the orbital motion.

Let us examine the figures for the different systems. For ζ Aurigae, the mass ratio measured by several experienced Mount Wilson astronomers on four plates varied from 1.3 to 2.2 (Christie and Wilson, 1935) with a mean value of 1.8. For 32 Cygni, Wright (1952) from measurements on two tracings found mass ratios of 2.6 and 3.0, but from the lack of internal agreement between the four lines measured on each tracing, he feels that the probable error may be 30 or 40 per cent.

For VV Cephei no measurements of radial velocity on the secondary spectrum have yet been made that can be used to give reliable radial velocities of the star. Goedicke (1939) has adopted a value of 0.7 as a compromise between obtaining unreasonably large masses and obtaining a reasonable fit on the mass-luminosity relation. For ε Aurigae, no conventional secondary spectrum is available to enable a mass ratio to be found.

For 31 Cygni, a number of measurements have been made at Victoria to try to obtain the mass ratio. Tracings of high-dispersion plates (4.6 A./mm.) have been used on which both primary and secondary spectra were measured against comparison lines by both Petrie and McKellar. Also six single-prism spectrograms have been measured in the conventional way by Petrie while, in addition, Wright has made differential measurements on high-dispersion tracings. The results are shown in the accompanying table from which it is seen that the direct measurements give a value of 3.5 or more while the differential measurement gives only

1.4. Miss Vinter Hansen (1944) from the Lick Observatory plates reports two values of the mass ratio 1.2 and 1.7.

TABLE II
MASS RATIO FOR 31 CYGNI

Source	No. of Plates	Mass Ratio	Author
Lick I-Prism	7	1.2	Vinter-Hansen
Lick I-Prism	7	1.7	Vinter-Hansen
Victoria Grating (4.5 A./mm.)	3	3.6	Petrie, McKellar
Victoria I-Prism	2	4.7	Petrie
Victoria I-Prism (quartz)	4	3.5	Petrie
Victoria Grating	7	1.4	Wright

One possibility that has been explored in a preliminary way is the photography of the spectrum into the ultra-violet to about 3000 A., to try to measure lines in the early-type spectrum where it is not badly blended. Spectrograms (16 A./mm.) have been secured for ζ Aurigae and 31 Cygni but, unfortunately, no well defined lines of the B-type spectrum have been found. To summarize, it is our belief that the determination of the mass ratio is not in a satisfactory state for any one of the ζ Aurigae stars. Further work is planned at Victoria.

It should be recalled that the uncertainty in the mass ratio is carried over into the absolute dimensions of the stars. Heights in the atmosphere of ζ Aurigae and 31 Cygni are commonly quoted to three significant figures, which is, we believe, quite unwarranted until the mass ratios are known much more accurately than at present. In this connection, the value of orbital inclination i is of importance. We should be pleased to hear from the specialists in interpreting light curves whether there is any prospect of being able to evaluate i for the ζ Aurigae stars.

Radial-Velocity Measurements on Chromospheric Lines. Radial-velocity measurements on the lines in the spectra of the primary stars of the systems ζ Aurigae, 31 Cygni and 32 Cygni well outside of eclipse and during total eclipse define the radial-velocity curves for these stars with a precision quite usual for binaries of similar spectral types. Just before and after eclipse, however, measurements on the chromospheric lines do not conform to this curve, but depart from it by amounts many times the probable errors of measurement. (McKellar and Petrie, 1952 and 1955; Wilson and Abt, 1954, and earlier references given in these papers). When first found about two decades ago, this effect was thought to arise from stellar rotation (Christie and Wilson, 1935). As measurements grew in number and in accuracy it became apparent that any contribution

due to stellar rotation must be very minor if present at all. Reasons for this conclusion include the facts that the deviations are far too random in nature and are often in the opposite direction to that required for rotation in the direction of revolution, and also they are probably too great in magnitude. The deviations, therefore, are most reasonably to be attributed to tangential components of mass motions of the gases in the chromosphere of the supergiant primaries and possibly contain both systematic and random contributions.

A reason for including this subject in the present paper is to urge that radial-velocity measurements be made at every opportunity on chromospheric lines in ζ Aurigae-like spectra. In this way it may be possible eventually to separate the systematic from the random effects and so to solve the problem of the origin of the force or forces causing any systematic motions.

The Widening Agency for the Chromospheric Lines. Ideally, for the application of the theory of radiative transfer to the interpretation of the spectrum of a star, we should like to have the star's atmosphere consist of a layer of gas whose physical parameters vary smoothly according to definite physical laws. There is good evidence from numerous sources that this ideal is not often realized, and the spectra of the ζ Aurigae-like stars provide some of the reasons for concluding that the atmospheres are inhomogeneous.

Among the lines of evidence that can be cited are: (1) the irregular deviations in radial velocity measured from the chromospheric lines, described in the previous section; (2) the occurrence of complex structure in the chromospheric K-line at certain times; and (3) the incipient structure indicated by the difference in shape of a given chromospheric line from time to time. The latter is illustrated by the scatter in a plot of equivalent width *versus* central depth for the chromospheric K-line in the spectrum of 31 Cygni. Thus, certainly in the outer chromosphere and probably in the inner chromosphere, the density is far from homogeneous.

In addition to the above points, less direct but very strong evidence has been put forward by Wilson and Abt (1954) that the chromosphere of the primary of ζ Aurigae must be discontinuous. Only by a model of this sort, having small comparatively dense condensations, could they reconcile the observed and calculated degree of ionization.

This subject has been introduced to emphasize the fact that the turbulent velocities required in accounting for the intensities of the chromospheric lines, and derived from curve-of-growth data, arise almost certainly from macroscopic as opposed to microscopic motions, and that

these large-scale motions must be kept in mind in the interpretation of data on spectral line intensities.

Spectrophotometric Study of the Chromospheric Spectrum of 31 Cygni. 1. The determination of the light ratio, $a = I_{CB}/I_{CK}$, (in Christie and Wilson's (1935) notation) is uncertain because, in the ultra-violet region where the B-type spectrum is strong, the K-type spectrum is so cut up by absorption lines that the continuum can rarely, if ever, be detected even with high dispersion. In the region to the red of λ4400 the contribution of the B-type star becomes small and therefore the intensities of chromospheric lines become uncertain. Over a 50 A. region at λ3925 Miss Underhill found $a = 2.3$, which is nearly the same as that found by Larsson-Leander (1956) from Michigan spectrograms. However, Wright and Lee (1956) determined $a = 0.77$ at λ3925 because they drew the continuum through high points on the tracings at λ3780 and λ4018. These values, of course, bear no relation to non-spectroscopic observations such as those of Guthnick, Schneller and Hachenberg (1935) for ζ Aurigae since the latter take insufficient account of line absorption.

2. The occultation of the B-type star by its giant companion can be studied spectrophotometrically by comparing regions where there seem to be few chromospheric lines. If f is the fraction of the light of the B star that has been cut off, fa may be found by comparing such regions with the same regions on plates taken far from eclipse and during totality. Plates taken within three days of totality at the 1951 eclipse of 31 Cygni were studied in this way and it was found that the time from first to second contact was just over two days, in agreement with preliminary photoelectric results discussed by Wood (1953).

3. Intensities of chromospheric lines can be found from intensity tracings, if a is known, by integrating the difference between the normal composite spectrum and the chromospheric spectrum for each line. This procedure assumes that differential continuous absorption introduced by the region producing the chromospheric lines is small. Miss Underhill has recently discussed this problem (1956) but, for 31 Cygni, we have not been able to separate the effect, if present, from the occultation effect. Victoria chromospheric intensities of lines in the region λλ3800–4000 have been compared with similar Michigan data, kindly supplied by Larsson-Leander, (1956) for plates taken on October 15, 1951. In spite of the different values of a, the Victoria intensities are, on the average, only about 25 per cent. larger. The different values of a are partially balanced by the position adopted for the continua to bring the resultant intensities into fair agreement.

4. Curves of growth for chromospheric lines in 31 Cygni have been

derived using the formula for pure absorption adopted by Wilson (1948) in his papers on ζ Aurigae. For Fe I lines observed within two weeks of totality, the data define the theoretical curves very well and the resulting temperatures, turbulent velocities and relative numbers of atoms are similar to those found by Wilson for ζ Aurigae. However, for other atoms the results are not as satisfactory. Lines of Ti I, V I, Ni I etc. cannot be measured until about three days before totality. Since these lines probably originate at levels different from the mean defined by the much stronger Fe I lines, it is doubtful whether the curves of growth for Fe I should be used to obtain the parameters for other atoms. As an example, Cr I lines from R.M.T. multiplets 1 and 2 at λ4260 and λ3730 do not seem to fit the Fe I curves of growth. The Cr I lines at λ4260 are strong and may correspond to positions on the curve of growth above the transition region. Lines such as Ca I, λ4227 and Ca II, λ3933 are even stronger. Since the derived abundance depends on the adopted curve of growth, the position of which varies with turbulent velocity and damping factor, it is readily understood that strong lines are particularly poor for studies of conditions in the chromosphere.

Victoria Observations of VV Cephei. The eclipse of VV Cephei was predicted to begin early in August 1956 and high-dispersion spectrographic observations were begun at Victoria late in March. The first strong plates showed apparent doubling of most of the well-developed chromospheric lines (Fe I, Cr I, Ca I, Sc II etc.). At first we linked the apparent duplicity with the observed double lines of Ca II observed in 31 Cygni by McKellar, *et al.* (1952) which they related to multiple condensations in the atmosphere. The much wider separation of the Ti II lines, λλ3759, 3761 and the somewhat lesser separation of the Ti II lines, λλ3900, 3913 were considered to be due to additional clouds high in the atmosphere. However, except for Ti II, most lines appear single after May 15. The Ti II lines λλ3759, 3761 changed about July 15 and the central portion now looks very much like the emission observed for RW Cephei by Merrill and Wilson (1956), and shown by them to be the reason for the apparent double lines in the spectrum of that star. It seems possible that both explanations for the shapes of the lines may be required when a detailed analysis is made, but at present we favour the emission hypothesis to explain the duplicities in the spectrum of VV Cephei.

Victoria Observations of ε Aurigae. In the present eclipse of ε Aurigae, first contact was predicted for 1955 June 5, second contact, December 14 and third contact 1956 November 8. To our knowledge, no photometric data have yet been published but the Victoria high-dispersion spectro-

grams give approximate confirmation of these dates. Struve (1953) and also Wright (1955) have pointed out that sharp redward components of lines such as Ti II, λλ4294, 4468 have been visible at least since 1954, and asymmetries were noticed in 1951. In the other systems considered here, such observations have usually been regarded as the beginning of atmospheric eclipse. The comparable Ca I line, λ4227, was visible in April and May, 1955, and was relatively strong from July to December but, like most of the other features in the spectrum, showed changes

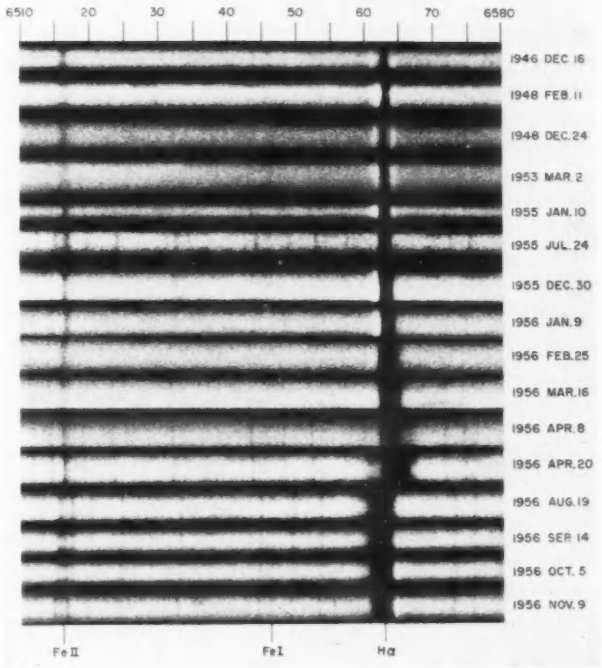

FIG. 1—Changes observed at Hα in the spectrum of ε Aurigae. Although the eclipse of ε Aurigae is total according to the light curve, large changes occurred in the Hα line both before and during totality. According to recent as yet unpublished observations of Gyldenkerne, first contact probably occurred about 1955 July 25 and second contact about 1955 December 1. Third contact was predicted for early November, 1956. The numerous sharp lines that vary in position in the spectra reproduced are terrestrial water-vapour lines. As well as the variations in Hα, real changes are seen to have occurred also in other spectral lines such as that of Fe II.

from plate to plate that probably indicate irregularities in the masses of gas through which the light passed. Strong sharp Fe I lines such as λλ3920, 3930 are quite marked from 1955 July 22 to November 25. They cannot be certainly identified from visual inspection of plates taken between 1955 December 15 and 1956 April 13 though, on some plates, the cores seem sharper than those observed early in 1955. Perhaps the most interesting and most revealing line in the spectrum of ε Aurigae is $H\alpha$ which has been photographed with a dispersion of 7 A./mm. As shown in the figure, prior to eclipse it exhibited emission features on each side of the central absorption. After first contact, an emission core at approximately the normal wave-length appeared and a strong, broad absorption line developed with, apparently, emission on the short wave-length side before mid-eclipse and on the long wave-length side after mid-eclipse. The above observations are qualitatively consistent with a system where the bright star passes behind a large mass of gas which is rotating in the same direction as the orbital motion.

In conclusion, we advocate in the spectrographic study of the ζ Aurigae-like stars, the use of the highest possible dispersion and resolving power.

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W URSAE MAJORIS TYPE SYSTEMS

By L. Binnendijk

Sixteen W Ursae Majoris type systems for which accurate data were available are discussed. These systems have variable periods. For eleven systems the time of principal minimum coincides with the middle of the increasing branch of the radial-velocity curve identified with the stronger lines. Then the component in front is the more massive and the cooler one, and has the greater intensity and larger size, so that we have an occultation. Changes in the form of the light curve occur in a number of systems.

A variable of the W Ursae Majoris type is defined here as having a light curve with curved maxima and almost equal minima, a period shorter than half a day and late spectral type. For practical observing reasons those systems are chosen for which the maximum light is brighter than the tenth magnitude and which are observable in the northern hemisphere.

Two dozen systems satisfy these conditions according to the Finding List for Observers of Eclipsing Variables. For eight of them, no photoelectric light curves or radial-velocity curves have been observed as yet. This leaves sixteen systems for which we have available the more accurate data we need in this discussion.

These systems have variable periods. In one case only (TX Cnc) no period variation is found, but this system has been under observation for only fifteen years. The period change of a number of longer observed systems is given with some recent observations included (figures 1 to 4). Small dots represent the normal points of visual observations, large dots the photographic data, and squares the photoelectric data. Open circles refer to results from Harvard patrol plates; crosses are from spectroscopic data.

The radial-velocity curves can be divided into two groups. In the first group (RZ Tau, ER Ori, AK Her, SW Lac) the time of principal minimum coincides with the middle of the decreasing branch of the radial-velocity curve identified with the stronger lines. This is the usual case for Algol and β Lyrae type stars. However, most systems belong to the second group (TW Cet, YY Eri, TX Cnc, W UMa, AH Vir, RZ Com, 44i Boo, V 502 Oph, VW Cep, AB And, U Peg). At the time of principal minimum the middle of the increasing branch of the radial-velocity curve identified with the stronger lines is observed (figure 5). For one system (V 566 Oph) no radial-velocity curves are available.

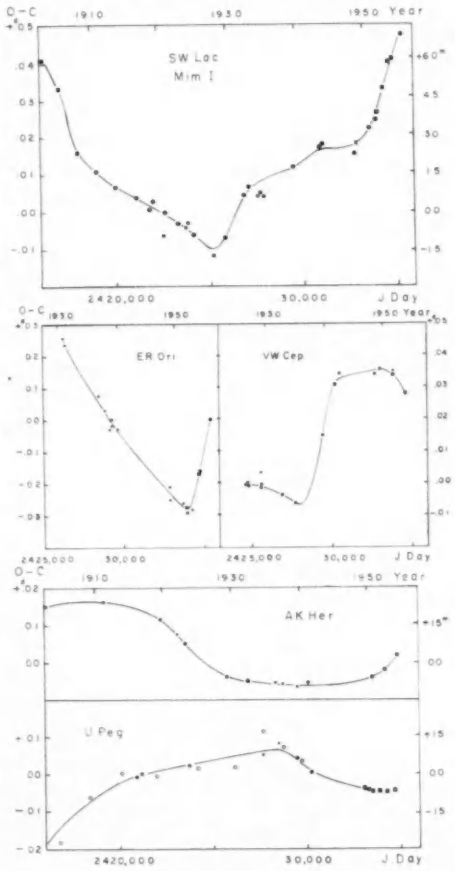

FIG. 1 (*top*)—Period variation of SW Lac. Visual normal points are small dots, photographic observations large dots, photoelectric data squares, and spectroscopic data crosses.

FIG. 2 (*centre*)—Period variation of ER Ori and VW Cep.

FIG. 3 (*bottom*)—Period variation of AK Her and U Peg. Open circles refer to results from the Harvard patrol plates.

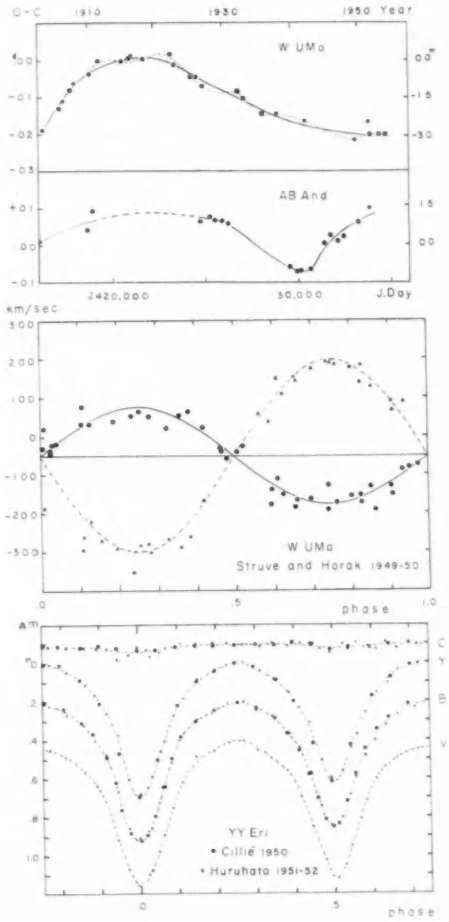

FIG. 4 (*top*)—Period variation of W UMa and AB And.

FIG. 5 (*centre*)—An example of the second group of radial-velocity curves (W UMa).

FIG. 6 (*bottom*)—The light curves and colour curves of YY Eri (V = violet, B = blue, Y = yellow, C = colour).

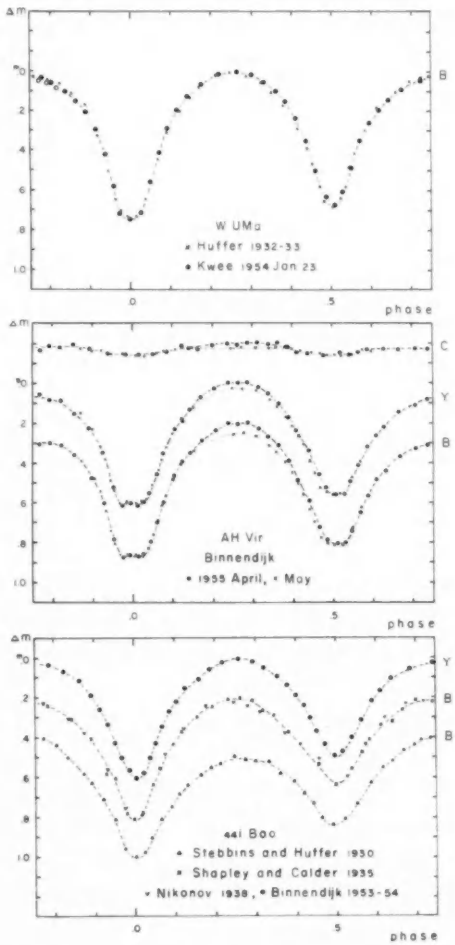

FIG. 7 (*top*)—The light curve of W UMa, showing that after one period the curve does not repeat itself.

FIG. 8 (*centre*)—The light curves and colour curve of AH Vir, showing variations in one month's time.

FIG. 9 (*bottom*)—The light curves of 44i Boo freed from the light of the bright visual component.

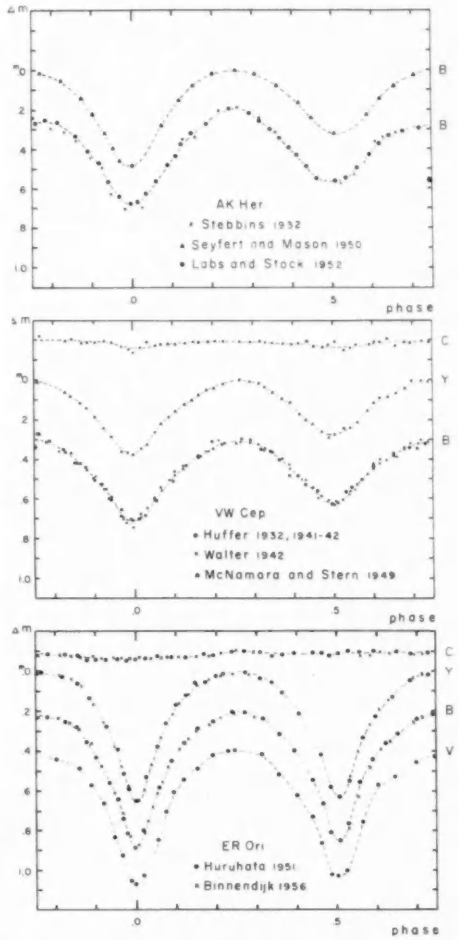

FIG. 10 (*top*)—The light curves of AK Her showing different shapes of the blue curves.

FIG. 11 (*centre*)—The light curves and colour curve of VW Cep.

FIG. 12 (*bottom*)—The light curves and colour curve of ER Ori.

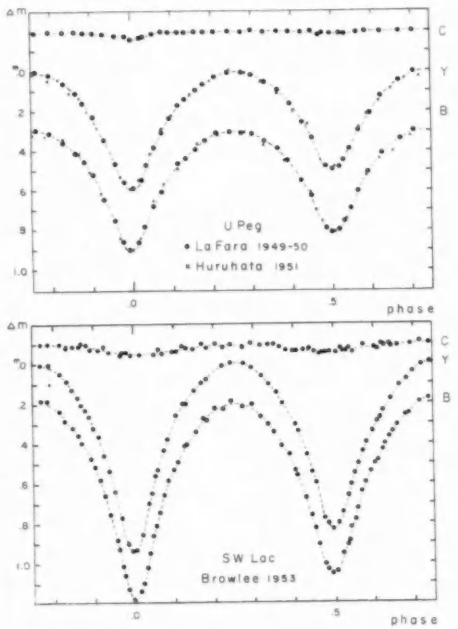

FIG. 13 (*top*)—The light curves and colour curve of U Peg found to be a flaring star.

FIG. 14 (*bottom*)—The light curves and colour curve of SW Lac.

Interesting conclusions can be made in the case of the second group. The radial-velocity curve associated with the stronger line has the smaller amplitude. The component in front during primary eclipse is the more massive one. The primary eclipse tells us that the component in front is also the cooler with smaller surface luminosity and later spectral type. This conclusion is correct for circular orbits and uniform light, or circular orbits and similar limb darkening. However, another deduction can be made which is not given in the literature as far as I know.

Consider the case for inclination $i = 90°$. We take the rotational ellipsoid with a,b,b axes as our working model. During both eclipses we have circular disks with radius b, and if we multiply the area πb^2 by the mean surface luminosity $\bar{I}$ we find the intensity l of the component. R. M. Petrie (1939) has found that for similar spectra the intensity ratio of the components equals the ratio of the line depths, or the ratio of the equivalent widths. We will use a simpler version here. A spectroscopist

refers to component one when he can see the corresponding sequence of absorption lines better than the other sequence. For similar spectra this means that the component in front (subscript 1) during primary eclipse has the greater intensity.

$$l_1 = \pi\, b_1^2\, \bar{I}_1 > l_2 = \pi\, b_2^2\, \bar{I}_2$$

But $\bar{I}_1 < \bar{I}_2$ at primary eclipse. The above inequality is then only possible when $b_1^2 > b_2^2$, or $b_1 > b_2$ because the negative sign has no sense here. This means that the component in front is also the larger one and that we have an occultation.

For $i = 90°$ the disks are elliptical with axes b and d. For an extreme case where $i = 70°$ and oblateness $\epsilon = 0.1$ one finds that $d = 1.013\ b$ so that the ellipses are very close to circles. Instead of b^2 we introduce bd in the inequality, and because in the working model the components are assumed to be similar the same result is found.

An independent check can be made using the shape of the minima. The orbital determination gives reliable results only for those systems for which the light curve is extremely accurate. Only in four cases can we make this check and they are all affirmative (YY Eri, W UMa, AH Vir, 44i Boo). The orbital elements were computed respectively by M. Huruhata and co-workers (1953), K. K. Kwee (1956), and the author (1955). This proves that the real star does not deviate much from our working model.

Z. Kopal (1955) recently came to a similar conclusion by considering the Roche constants of the systems derived from the orbital elements. Because of the difficulty of finding reliable elements for these systems his approach is statistical in nature. His conclusion is that the primary minima are likely to be occultations. We can be more definite about the eleven systems but can make no conclusion about the other systems.

Several available photoelectric light curves in several colours were compared together with their colour curves. A number of them are reproduced here in figures 6 to 14. No changes were made in the magnitude scale. During both eclipses we look at the cooler and redder ends of the oblate components. We observe that the colour of the system is redder at both eclipses as a consequence of the gravitation and reflection effects.

The light curves of these systems often show small changes with time (W UMa, AH Vir, 44i Boo, AK Her). The flare phenomenon is more serious in the ultra-violet than in the yellow (U Peg). The more common and more regular up and down motion of the curved maxima seems to be caused in a similar way. There is an indication that during the time an extremely high maximum occurs, the colour of the system is bluer than during a normal maximum (AH Vir).

More observations of this kind would be very useful. It is proposed that several photoelectric observers in different longitudes and a spectroscopist concentrate their efforts on a promising system such as AH Vir. It is found, for example, that the up and down motion of the maxima is connected with a shift in the position of the secondary minimum. In two cases complete eclipses are observed (W UMa, AH Vir) and valuable information can be derived about the amount of light caused by the gas streams around such systems.

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EVOLUTION OF CLOSE BINARY STARS

By Su-Shu Huang

Introduction

In order to understand the nature of close binaries, we emphasize that a general probing of many plausible models is more important than a detailed calculation based upon a definite model, at least for the time being. For example, a good fit of the light curve to an elaborate model of the binary star has very little physical meaning, because it could simply be due to the many adjustable parameters necessarily associated with the elaborate model. Instead, we should build, for each system, a model directly from the results of photometric and spectroscopic observations. The model should then be examined in the light of its dynamical feasibility. Next, we should investigate whether there is any general relation among those dynamically feasible models for different binary systems, and if possible, isolate some parameters which control the physical and dynamical state of the binary system. In other words, we should build up models for close binary systems gradually from the photometric and spectroscopic results, as well as from dynamical and statistical considerations in the manner of successive approximations.

Mass Transfer in Binary Systems

The study of mass transfer in binary systems, which has far-reaching consequences in the problem of stellar evolution, was first introduced into astrophysics through the spectroscopic studies of β Lyrae 15 years ago at the Yerkes Observatory. The model for β Lyrae derived by Struve (1941) from spectroscopic observations prompted a theoretical study by Kuiper (1941) of the superficial instability of the component stars in the system. There are two main contributions in Kuiper's paper: (1) In treating problems of gaseous flow in a binary system, the importance of the zero-velocity surfaces passing through three Lagrangian points on the common axis of the stars is emphasized. For brevity, Kuiper calls these the inner and outer contact surfaces. (2) Two kinds of ejection have been proposed: Type A denotes ejection at the inner Lagrangian point which transfers mass from one component to another, and Type B denotes ejection at one of the outer Lagrangian points through which mass from the unstable component escapes into outer space.

If β Lyrae were the only star which shows gaseous streams in the system, the problem of mass transfer would never become an important topic of astrophysical research. Systematic spectroscopic studies at the

McDonald Observatory and elsewhere show that the velocity curves of many eclipsing binaries are distorted by gaseous streams in the systems (Struve, 1949). Distortion of the velocity curves by gaseous streams explains two well-known phenomena: (1) the discrepancy between the values of $e \cos \omega$ determined from the light curve and from the velocity curve, and (2) the biased distribution of ω of spectroscopic binaries—a phenomenon known as the Barr effect (1908). Another evidence of the existence of gaseous streams is, of course, the occurrence of emission lines in many eclipsing binaries.

In the meantime, photometric studies show that the secondary components of several eclipsing binaries have already reached the size of the inner contact surface, and are therefore in a state of superficial instability (Wood, 1946). Combining the results obtained from both spectroscopic and photometric studies we can conclude that in many close binaries there exists a gaseous stream resulting from the overflow of mass of the superficially unstable component through the inner contact surface. This stream of gas is the agency of mass transfer in binary systems. If it persists for a long period of time, its effect will show up in two ways: (1) a change in the orbital elements of the dynamical system, and (2) a change in the internal constitution of the component stars.

Theory of Emission Rings

Having found that the emission lines of RW Tau consist of two components and undergo eclipses, Joy (1942) proposed the existence of a tenuous gaseous ring in the equatorial region around the primary B9 component. The ring is supposed to rotate in the same sense as the orbital motion. In this way, Joy was able to explain beautifully the behaviour of emission lines during the eclipse. Further investigations (Struve, 1946; McKellar, 1950) show that emission lines found in most early-type eclipsing systems can all be explained in terms of a gaseous ring.

We can show that the ring formation is a necessary consequence of the fact that the secondary component which ejects matter has a considerably smaller mass than the primary (Struve, 1948). Let us imagine that we take a unit mass from the secondary and add it to the primary without affecting the orbital motion. We see immediately that this is impossible unless we make the added matter rotate around the primary. If

$$\mathcal{M} = \frac{m_1 m_2}{m_1 + m_2} \frac{2\pi a^2 (1 - e^2)^{\frac{1}{2}}}{P} \tag{1}$$

is the total orbital angular momentum of the system, where a is the semi-major axis of the relative orbit and others have their usual meanings, the

angular momentum per unit mass of the primary and per unit mass of the secondary will be

$$h_1 = \frac{m_2}{m_1 + m_2} \frac{\mathscr{M}}{m_1} \quad \text{and} \quad h_2 = \frac{m_1}{m_1 + m_2} \frac{\mathscr{M}}{m_2}. \tag{2}$$

Therefore the extra angular momentum per unit mass that a transfer of mass from the secondary to the primary will cause is

$$\Delta h = h_2 - h_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) \frac{2\pi a^2 (1 - e^2)^{\frac{1}{2}}}{P}. \tag{3}$$

This supplies the angular momentum for the gaseous ring. We have assumed that by the removal of mass, no change in motion is experienced by the secondary. We define this as ejection along the common axis. For the frontal ejection, the ejected mass takes away from the secondary an angular momentum in excess of its average share. Thus, for the frontal ejection, Δh will be greater than that given by (3).

Let the gaseous ring be a circle of radius a_{em}. We assume also that the ring is in the plane of the orbital motion. If V_{em} denotes the observed rotational velocity of the emission ring, the angular momentum of the ring per unit mass will be $(a_{em}\, V_{em})/\sin i$ which, after equating to Δh given by (3), gives

$$\frac{a_{em}}{a} = \frac{(m_1 - m_2)}{m_2} \frac{K_1(1 - e^2)}{V_{em}}. \tag{4}$$

We should note that a slight change in the definition of V_{em} has been made here in order to simplify the formula,

$$\frac{a_{em}}{a} = \alpha(\alpha + 1) \frac{K_1^2(1 - e^2)}{V_{em}^2}, \tag{5}$$

which has been derived in a previous paper (Huang and Struve, 1956). In (5), α denotes the mass ratio, m_1/m_2.

Combining (4) and (5) we obtain

$$\frac{\alpha - 1}{\alpha(\alpha + 1)} = \frac{K_1}{V_{em}}. \tag{6}$$

For frontal ejection we should replace the equality sign in (6) by a "smaller" ($<$) sign. It can easily be seen that the maximum value of $(\alpha - 1)/[\alpha(\alpha + 1)]$ is $\sqrt{2}/(4 + 3\sqrt{2}) = 0.172$. Thus, if K_1/V_{em} is greater than this value the ring must have been formed by gases from frontal ejection. In Table I we have listed both $(\alpha - 1)/[\alpha(\alpha + 1)]$ and K_1/V_{em} for all stars for which V_{em} is available (Struve, 1946; McKellar, 1950). It

TABLE I

ANGULAR MOMENTUM OF THE GASEOUS RING

Star	α	$\frac{\alpha-1}{\alpha(\alpha+1)}$	$\frac{K_1}{V_{em}}$
RX Cas	—	—	.240
SX Cas	—	—	.333
VW Cyg	3.4	.160	.117
RY Gem	4.5	.141	.135
TT Hya	3.0	.166	.163
AQ Peg	4.1	.148	.121
RW Per	5.6	.125	.082
RW Tau	3.5	.159	.157

is interesting to note that the two quantities have the same order of magnitude, as should be expected. In the case of RX Cas and SX Cas, we conclude that the ejection of mass by the unstable component star takes place in the front. On the other hand, we cannot conclude that rear ejection takes place in the other systems, because a small part of the ejected matter may have escaped into outer space carrying away a greater part of the angular momentum.

There are several empirical reasons which support the ring hypothesis. Assuming gravitational interaction between the primary and the gaseous ring alone and neglecting the perturbation by the secondary, we can derive the empirical result:

$$V_{em}^3 \propto P^{-1} \tag{7}$$

obtained by Struve (1946).

Since we are able to estimate an upper and a lower limit of the size of the ring from the nature of the eclipse of emission lines, limits of the mass of the central star can be derived from V_{em}. A simple calculation shows that the limits are in agreement with the mass that would be expected from the spectral type and luminosity of the star (Huang and Struve, 1956).

In this paper it is shown also that all systems in which emission lines appear have their secondary components in touch with the inner contact surfaces. On the other hand, systems in which the secondary component fills up the corresponding lobe of the inner contact surface do not all show emission lines. This suggests that filling up of one lobe of the contact surface by the secondary component is not a sufficient condition for forming a semi-stable ring around the primary component. Empirical evidence seems to indicate that formation of a semi-stable ring depends also upon the size of free space between the primary and the correspond-

ing lobe of the contact surface. In order to form a ring, it must be larger than a certain critical size. If the free space is too small, no semi-stable ring could be formed and emission lines appear only as long as the subgiant component ejects matter. Since the ejection, like the prominences, may not be a steady process, we should not expect the emission feature to be a permanent phenomenon. Indeed, in RW Tau (Wyse, 1934; Joy, 1947) and U Sge (McNamara, 1951) emission lines sometimes disappear completely.

Secular Effect of Mass Transfer on the Orbital Elements

At its simplest, this is a three-body problem. Actually it is more complicated than this, because the gaseous particles collide with one another and moreover the stars have finite radii. Consequently, a satisfactory solution to this problem is beyond our means. Rough calculation shows (Huang, 1956), however, that the period of the binary system can either increase or decrease, depending upon the relative amounts of mass and angular momentum which escape to outer space with respect to those which are absorbed by its companion. These fractions are obviously related to the degree of violence of ejection and most likely vary with time. Thus, the change in period will be erratic rather than systematic. Since a decrease in the separation of the two components of a binary system favours the formation of a common envelope which in turn facilitates the escape of mass to outer space and therefore increases the separation, an equilibrium state exists in which the major axis remains unchanged (or at most fluctuates around the stationary separation). This may actually happen in all close binaries which have common envelopes such as in the W UMa stars. In the case of β Lyrae, the period has been increasing for the past centuries, but the rate of increase in period is definitely slowing down.

Secular Effect of Mass Transfer on Evolution of the Component Stars

No computation has been made, although from physical plausibility we have suggested (Huang, 1956) a sequence of unstable stars covering Population II Cepheids and cluster-type variables as one extreme and novae as the other extreme, with stars like SS Cyg, etc. as intermediate objects. They are supposed to be caused by the same kind of instability that develops in the core of a star in its later stage of evolution. Several predictions by this hypothesis seem to agree with the observations.

Most recently Crawford (1956) has suggested that if the superficially unstable star has a white dwarf companion, the accumulation of ejected hydrogen atoms over the surface of the white dwarf could lead, according to the Mestel mechanism (1952) to a supernova of Type I. The

unstable star provides the hydrogen gas which cannot otherwise be obtained in the Population II systems where supernovae of Type I have been observed.

Formation of Binary Stars

Formation of binary systems through star capture in a resisting medium has been studied. In an unpublished paper we have found that the frequency of binary formation is inversely proportional to the fifth power of the velocity dispersion of the stars in a group. Thus, a simple calculation can readily show that the probability for star capture in the solar neighbourhood is too small to account for the high frequency of binary stars. We suggest that binary stars have been originally formed in star associations during their formative period. For example, the W UMa stars whose occurrence is high among binary stars (Shapley, 1948) might be formed in T associations.

I am greatly indebted to Professor Otto Struve who, through his stimulative discussion and generous encouragement, has initiated me into the field of double-star astronomy.

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THE FISSION THEORY AND THE ORIGIN OF BINARY SYSTEMS

By G. J. Odgers and R. W. Stewart

The Classical Fission Theory. According to the fission theory the origin of binary stars is regarded as due to the break-up of a single mass by its rotation. Darwin and Jeans considered a general secular increase in angular momentum with the density remaining uniform since this is equivalent to constant angular momentum with gradually increasing density. This latter circumstance was thought of as arising from a gravitational contraction of the original mass, and viscosity was supposed to be great enough to keep the angular velocity constant throughout the mass. With small values of the angular momentum the mass takes the form of an oblate spheroid; as the angular momentum increases, this form becomes unstable and an ellipsoid with three unequal axes develops which is stable initially. As the angular momentum increases further, the longest diameter, which is perpendicular to the axis of rotation, elongates until it is almost three times the length of the shortest diameter and this form is then secularly unstable and does not pass into any other secularly stable figure. Two sorts of stability must be distinguished, namely secular and ordinary stability. The former may be thought of as long-term stability and if a system is secularly stable it is also ordinarily stable, or possesses short-term stability. If ordinarily unstable a system is also secularly unstable, but a system which is ordinarily stable may or may not be secularly unstable. Poincaré showed that the ellipsoids with three unequal axes developed a furrow at one end when they became secularly unstable, and Liapounoff and Jeans then demonstrated that the new figure was secularly unstable so that there was no way of storing the angular momentum except by fission. However, in order that fission into two detached masses moving in an approximately circular relative orbit could take place, it was necessary to assume ordinary stability, for otherwise the changes would not be slow and viscosity would not be sufficient to maintain uniform angular velocity. This assumption was shown to be false by Cartan who proved that when the ellipsoids became secularly unstable they also became ordinarily unstable, and hence fission in the sense of binary star formation is impossible by this process.

Although the fission theory in its old form may be considered invalid the proof does not carry over to discrediting subdivision theories in general. The classical theory was developed under very special assump-

tions of uniform density and constant angular velocity (rigid-body rotation), i.e. infinite viscosity and no shear stresses.

It may also be noted that the spheroids and ellipsoids of the classical theory were obtained by purely equilibrium considerations.

However, the total angular momentum $\boldsymbol{H}$ must remain constant along an evolutionary sequence as must the total energy $V + \frac{1}{2} \mathcal{I}\omega^2$ where $\mathcal{I}$ is the moment of inertia. The classical forms do not satisfy these conditions and do not form a possible evolutionary sequence.

In any configuration the Virial Theorem states that

$$\frac{1}{2}\frac{d^2\mathcal{I}}{dt^2} = 2T + V,$$

if a steady state is ultimately reached $2T + V = 0$, but $T + V = h$ the total energy; so $T = -h$, with $\boldsymbol{H}$ remaining constant. It seems likely that it is impossible to store this amount of kinetic energy and angular momentum in a single mass so that the kinetic energy in an orbit and orbital angular momentum must absorb most of it. For instance, in the solar system about 98 per cent. of the total angular momentum lies in the orbital motion of the major planets; one may also refer to the empirical fact that stars do not, and presumably cannot, store much angular momentum.

Gravitational Instability and Shear. Instead of the mechanism of rotational fission of the old sort, let us consider the problem of a contracting gas and try to find what effects gravitational instability and turbulent transfer of mass and momentum might have. The classical theory was a detailed, purely deductive theory of a very special type of stream-line flow, namely, pure rigid-body rotation. A very important and fundamental difference between stream-line and turbulent flow is the ability of the latter to transmit large shear stresses and to diffuse heat and matter very quickly. This may be considered as being due to the effect of large-scale eddying motions whose rapid fluctuations transport mass and heat quite quickly. Such motion requires a continuous supply of energy for its maintenance and this is provided by the agency of the mean flow working against the turbulent stresses. In this case, that of a mass of gas at low density contracting gravitationally the chief contribution to the mean flow comes from the contraction and the energy maintaining the turbulent flow is taken from the gravitational potential energy. Since the turbulent flow helps transport the mean flow itself, no method treating the motion as a small perturbation can succeed; there will be an intimate relation between the supply of energy to the turbulent motion and the transport of momentum. It is easy to show that motion on a cosmic scale

must be turbulent and we have to try to combine the dynamics of such motion with the operation of gravitational contraction.

In any gas which has internal movements the pressure and therefore the density will not be constant. Considering a region of higher than average density we find in general that the pressure is higher than average, thus tending to disperse the concentration. However, gravitational attraction caused by the matter within the region tends to hold it together. This attraction is very small except for very large masses of cosmic dimensions, but on such a scale there will be a critical size of concentration such that the pressure gradient is exactly counterbalanced by the gravitational attraction. For all sizes greater than this critical size the force of gravity dominates and the concentration will not disperse.

For a gas of given composition its physical properties are largely determined by any two of the three parameters density, temperature and pressure, although other influences such as the nature and intensity of external radiation and the violence of internal motions are not completely negligible. If for the moment we ignore such other influences we can use a dimensional argument to find the behaviour of the critical size.

Suppose that the gas behaves as a perfect gas, then the relevant parameters are density ρ gm. cm.$^{-3}$ temperature T (°K.), the gravitational constant G (erg. cm. gm.$^{-2}$); the gas constant $\mathscr{R}$ (erg. mol.$^{-1}$ K.$^{\circ -1}$) and the molecular weight μ (gm. mol.$^{-1}$).

Dimensional analysis then leads to the results that the critical size

$$\lambda(\text{cm.}) \propto \left(\frac{\mathscr{R}T}{G\mu\rho}\right)^{1/2} \tag{1}$$

and the critical mass

$$m(\text{gm.}) \propto \left(\frac{\mathscr{R}T}{G\mu}\right)^{3/2} \rho^{-1/2} \tag{2}$$

By considering the density and pressure fluctuations it can be shown that the constant in (1) is $(3\gamma/4\pi)^{\frac{1}{2}}$, γ being the ratio of specific heats. We have then that

$$\lambda_0^2 = \frac{3}{4\pi}\frac{c^2}{G\rho}.$$

From the kinetic theory of gases it is known that

$$P = \tfrac{1}{3}\rho\overline{V^2}$$

where $\overline{V^2}$ is the mean of the squared speeds of all the particles in the medium; now λ_0^2 is proportional to $\overline{V^2}$ which is proportional to T so that a sudden cooling reduces λ_0; if the cooling is so sudden that the linear

dimensions cannot keep pace, subsidiary condensations form at distances apart of the order of the new value of λ_0. On the other hand, if cooling takes place sufficiently slowly contraction can keep pace with it and subsidiary condensations will not occur. This may be a clue to the origin of multiple systems and star clusters; some at least of binary and higher systems could be considered, on this view, as remnants of the original clusters. Whether or not sudden cooling could have taken place—by sudden is meant cooling rapid enough to outpace the very slow pace of gravitational contraction—depends on the initial conditions. For instance, a sufficiently isolated mass can lose energy by radiation rapidly enough for this purpose: association of hydrogen atoms to form molecular hydrogen would drastically reduce the number of particles and decrease the heat energy allowing smaller subsidiary condensations to form. This process is forbidden as a two-body collision (the quantum theory angular momentum rules require collisions of three bodies which is too infrequent a process to be of any importance) but the two-body collision and association is a possible process if dust particles are present. On this view then the process of gravitational instability and subsequent contraction to form a star can lead, depending on the initial conditions, to a multiple system. The attempt to follow the development of the mass when secondary centres of contraction occur is much more difficult since mutual interaction cannot be ignored. Using the above result in the form

$$\lambda_0^2 = \frac{3}{4\pi} \frac{\gamma \mathcal{R} T}{\mu G \rho},$$

consider a medium of atomic hydrogen $\gamma = 5/3$, $\mu = 1$, at a density of $\rho = 10^{-22}$ gm. cm.$^{-3}$ The critical mass is then $2.88 M_\odot T^{3/2}$, and the critical radius is $78\ T^{1/2}$ parsec.

Now the negative potential energy, the work done by gravitation in drawing together the material from a state of infinite diffusion, is given by

$$G \int_0^M \frac{M_r}{r} dM_r = \Omega$$

and in the case of uniform density it is found that this equals

$$\frac{3}{5} \frac{GM^2}{R}.$$

The heat energy is $c_v T$ per unit mass, so that the total heat energy is H given by

$$H = c_v \int T \rho \, dv$$

$$= c_v M \bar{T}$$

and since $$c_p - c_v = \frac{\mathscr{R}}{\mu}, \frac{c_p}{c_v} = \gamma$$

$$c_v = \frac{\mathscr{R}}{\mu(\gamma - 1)}.$$

The mean temperature is given by

$$\bar{T} = \frac{\mu}{\mathscr{R}} \frac{1}{5} \frac{GM}{R}$$

so $$H = \frac{\Omega}{3(\gamma - 1)},$$

hence even with an initial temperature of 1° K., and hence with an initial radius of $3.44 \times 10^7 R_\odot$, a contraction to $\sim 3.44\ R_\odot$ involves a 10^7 fold increase in T i.e. a temperature of 10^7° K., sufficient to liberate nuclear energy and transform the contracting mass into a star.

Origin of binary systems

The close binary systems of short period would have to be considered not as having arisen from fission but by dynamical evolution from wider pairs, the motion taking place in a resisting medium. As is well known, the effect of this is continually to decrease the semi-major axis, and hence to decrease the period and if the resistance depends on a power of the speed greater than the first, the effect is to decrease the eccentricity steadily also. The orbital angular movement decreases also, being transferred to the medium and to the rotational momenta of the components.

With the fission theory (prototype β Lyrae taken by Jeans) increasing contraction gave rise to tidal couples and Darwin showed that qualitatively such "tidal friction" produced: (i) increasing separation; (ii) increasing period; (iii) increasing eccentricity.

However, when components are at distance r apart the tides raised by either on the other are proportional to $1/r^3$ and the tidal couple to $1/r^6$ so that the effect decreased very rapidly and could not lead to wide pairs. They were reduced then to the supposition that binary systems fell into two classes:

(1) systems formed by fission, periods short but lengthening,

(2) systems formed by condensations; in Jeans' words: "long-period binaries would seem to represent the remains of independent condensations in the parent nebula which failed to get clear of one another's gravitational fields and have been describing orbits about one another

ever since, their periods being on the average continually lessened by encounters with other stars". According to Darwin's analysis relating to tidal friction the former class have periods < 55 days, the latter > 55 days. However, there is no bifurcation in binary star statistics of this sort and it is suggested that binaries are all in class (2). The important difference with Darwin and Jeans' class (2) comes from the consideration that centres of condensation over one parsec apart would take a quite impossibly long time to evolve into a close pair. We have to look for centres of condensation closer than indicated on the static model. We have seen that the original mass may subdivide under certain circumstances; in a sense, therefore, though rotational fission has been shown to be impossible, a sort of turbulent fission may take place; this turbulent motion gives rise to much larger density fluctuations than were contemplated by Jeans. These in turn are amplified by the process of gravitational instability and it depends on what stage in the development of the original contracting mass that these fluctuations occur whether we shall have a close system or not. To examine this process in more detail we have to consider the "internal motions in a contracting gas". The observed fact that it is hoped to explain is that the distribution function of the semi-major axis of binary orbits is approximately Gaussian, and there is no evidence for two distinct classes of binaries; also consideration of magnitude differences shows that the mass ratios are similar in the close and wide systems; again rotational fission is ruled out on dynamical grounds alone. It will be shown that the mechanics of turbulent motion in a contracting gas are such that independent condensations in more or less random positions within the original mass can occur, depending on the amount of the total angular momentum, if large no stars, if very low one star. If a ternary or higher system is formed initially, its subsequent development depends on the processes of 3-body or n-body motion and it is quite possible that one body is ejected if the original distances are all comparable. It is only where the system forms binaries two-by-two, so to speak, i.e. in 3-body system one distance very small with respect to the other, that the system remains stable. For example, Castor, a sextuple system, is stable, whereas the Orion Trapezium is probably unstable. Important attempts to make use of the theory of turbulence (in problems of this sort) have been made by von Weizsacker and by Kuiper. This theory advances rather rapidly, however, and it appears that the ideas used by them must now be definitely regarded as incorrect.

Von Weizsacker imagines that around a condensation giving rise to a gravitational field a set of highly organized eddies develop in shells, the

shearing forces being minimized by rotations whose direction alternates in a sort of ball-bearing effect. Much attention has recently been devoted to the development of turbulence under shear (which is what is required in this problem). A great deal was previously known about the effect of rotation on condensation but almost nothing about the effect of shear and Townsend in his recent book on this topic shows that in shear flow the most active eddies are not oriented in a ball-bearing sense with axes normal to the direction of flow, but with axes essentially in the direction of the flow itself, and they act not to minimize but to maximize the shearing stresses. The long-lived stable motions envisaged by von Weizsacker are then very different from those observed experimentally.

Kuiper in his theory of the origin of the solar system uses a modification of von Weizsacker's argument and invokes the Kolmogoroff spectrum of turbulence (energy distribution with respect to eddy size). However, it is now thought that in real turbulent fields the eddies which contribute to the Kolmogoroff region of the spectrum contain only a negligible proportion of the turbulent energy. The energy containing eddies are known to be quite differently distributed and their spectrum depends—and this is important for our problem—on the initial conditions, i.e. on the nature of the disturbance which created the turbulence; a detailed account of these large or active eddies is difficult therefore to work out. Kuiper also requires eddies to condense eventually into stars or planets, but most turbulent eddies are short-lived unstable structures with life-times of only a characteristic revolution. There are exceptions—a smoke-ring, for example—which lasts for many revolutions but these are all characterized by a low-pressure nucleus and are the last places to expect the onset of gravitational instability.

To examine the role of turbulence in stellar formation consider first a volume of gas contracting in its plane of rotation. Conservation of angular momentum requires an increase in angular velocity and the associated linear velocities. If there is a linear change of size of a factor q in the plane of rotation the moment of inertia changes by a factor q^2 and therefore the angular velocities must change by q^{-2} and typical linear velocities by q^{-1}. The linear velocities thus increase in proportion to the decrease in size.

If in addition to an overall rotation the gas has other less organized internal motions, these also will be intensified by a contraction even if the internal motions are irrotational. Regardless of their nature the disorganized internal motions impose Reynolds' stresses on the gas; these stresses may or may not have shear-type components $\overline{u_i u_j}$, $i \neq j$, but they

will always have normal components $\overline{u_i}$ which behave almost exactly like pressure and when the gas contracts, work must be done against these stresses, such work appearing as increased kinetic energy of the disorganized motion. Hence the collapse of a gas tends to increase the intensity of the internal movements; together with increase in density this tends to increase the magnitude of pressure and density fluctuations within the gas.

If we start with a gas of cosmic dimensions this will be in a state of turbulence (because of the high Reynolds' number vD/ν; ν is the kinematic coefficient of viscosity) so that density fluctuations must exist and if the total angular momentum is not too great there will always be some sufficiently large to enable gravitational contraction to commence. This will be resisted by two forces: pressure and the normal Reynolds' stresses due to the turbulence (the total normal stress is $P + \rho\,\overline{u^2}$. During collapse, work is done against these forces resulting in an increase in the turbulent energy which becomes intensified and as the density increases further centres of subdivision can arise. This is really fission again in a new guise—turbulent or non-rotational fission. The old fission theory postulated uniform density and infinite viscosity, to provide uniform angular velocity. (However, the motion *must* be turbulent, with actual velocities and viscosities, rigid body rotation is itself unstable). A rough physical picture would be that the turbulent motion transfers angular momentum from the centre to the outside of the contracting mass giving rise to new subdivisions.

Even if a density fluctuation is too small to be stable and dissipates, something has been gained, for at the peak of the fluctuation the temperature is increased and energy radiated so that some of the internal energy of the gas in this region is lost and when the gas expands to the ambient pressure its density will remain higher than average. If this portion of the gas is again compressed at a later time by other turbulent motions, it will be in a more favourable state to become gravitationally unstable.

Theoretical Model of Star Formation

For further progress some sort of model of the system has to be contrived. A simple case which demonstrates the important features is obtained if we consider a gas contracting towards a point with velocity proportional to the distance. Suppose there is rotation and disorganized motions averaging zero when averaged in some convenient way. Using Cartesian tensors, with 3-axis, axis of rotation

$$v_i = -\alpha x_1 + \beta\epsilon_{ih3}x_h + u_i$$

v_i velocity in i-direction, α independent of x_i; $-\alpha x_i$ is local contraction velocity component.

β function of $x_1^2 + x_2^2$ only; $\beta\epsilon_{ih3}\, x_h$ is local velocity component due to rotation.

ϵ_{ih3} is the isotropic tensor (generalization of δ_{ij})
$= 0$, if i, h, 3 not all different
$= 1$, if i, h, 3 in cyclic order
$= -1$, if i, h, 3 in anti-cyclic order.

The equations of motion are

$$\frac{\partial v_i}{\partial t} + v_j \frac{\partial v_i}{\partial x_j} + \frac{1}{\rho}\frac{\partial P}{\partial x_i} = g_i$$

$q^2 = \sum_i u_i^2$ is taken as a measure of the total kinetic energy of the disorganized motion we can show that

$$\frac{\overline{\partial q^2}}{\partial t} = 2\alpha\overline{q^2} + \alpha\overline{x_j \frac{\partial q^2}{\partial x_j}} + \beta\epsilon_{ij3}\,\overline{u_i u_j} + \beta\epsilon_{jk3}\,\overline{x_h \frac{\partial q^2}{\partial x_j}} + \overline{u_j \frac{\partial q^2}{\partial x_j}} + \overline{\frac{u_i}{\rho}\frac{\partial P}{\partial x_i}} = 0;$$

where the various terms may be interpreted physically as follows:

3rd term = interaction of rotation and Reynolds' shear stress
2nd term = transport of energy from place to place by contraction
4th term = transport of energy from place to place by rotation
5th term = transport of energy from place to place by turbulent motion
6th term = transport of energy from place to place by pressure fluctuations
1st term = interaction of contraction with the normal (pressure like Reynolds' stresses) and is positive.

Ordinary and second viscosity tend to downgrade the kinetic energy to heat and in slow collapse may dissipate disorganized motion. But in many cases the 1st, 5th and 6th terms may dominate motion and give rise to the turbulent fission described above. It is expected that the detailed development and analysis of the above model will be published in the near future.

SUMMARY

By Otto Struve

Let me start by saying that I have never attended so interesting and fruitful a scientific conference as the one which is now coming to its end. The National Science Foundation, the Dominion Astrophysical Observatory and Dr. Petrie's steering committee are to be congratulated upon their wisdom in making this meeting possible and in organizing it along the broadest possible lines.

We have listened to a series of excellent papers on astrometric, photometric, and spectroscopic problems of double stars, and this morning we were exposed to two important theoretical discussions of the formation and evolution of binaries.

I have asked the members of the conference to give me their individual comments, and I shall incorporate them in my remarks. The most concise and comprehensive statement comes from Dr. Kron: "I think that double-star astronomy needs more of an approach like Dr. Huang's". With this appraisal I most heartily agree, not because Huang is one of my colleagues at Berkeley (and I might remind you that there has been no collusion between Lick and Berkeley, and that Mr. Kron's remark was entirely spontaneous), but because I believe that the observational data at such a conference should lay the basis for discussions of origin and evolution.

But a solid foundation of facts we must have, in order that the theoretical workers have something to discuss; and this leads me to the following comment: observational results are accumulating far too slowly. Since we cannot often predict which stars are likely to furnish the most valuable clues, we must of necessity observe a great many. We have also concluded that there are many "abnormal" double stars, and few, if any, that can be described as entirely "normal"; hence, we must observe the same stars over and over, and we must, sooner or later, embark upon as nearly continuous a series of observations as is possible of such remarkable "abnormal" systems as ϵ Aurigae, VV Cephei, β Lyrae, U Cephei, and perhaps a dozen others. It seems to me that the theoreticians even now are greatly hampered in their work by the lack of reliable observational data; and this conference confirms in my mind an estimate of our goal in observational astronomy which I communicated some months ago at the request of the National Science Foundation: we should aim toward an increase by a factor of ten in the flow of observational data

during the next 25 years. This requires more telescopes and accessory instruments of the type now contemplated for the National Observatory of the United States and of the type recommended in the resolutions passed earlier today. It also requires the training of more astronomers in all fields. I might add that it will take even more time and more money to accomplish the latter task than to procure the instrumental facilities.

I had the impression, while listening to the papers at this conference, that the astrometric experts are under the impression that their work is not sufficiently appreciated by the astrophysicists. They have called attention to the serious lack of observers of visual double stars, of mass ratios, of proper motions and of parallaxes, and they have voiced the opinion that more young astronomers should be induced to devote themselves to these tasks. There is no doubt that most young scientists prefer to work in newly-opened fields of research. Even some of the older types of astrophysical research, such as the measurement of radial velocities, fail to attract the younger astronomers.

Under our system it is difficult to cope with this problem. We cannot compel our students to specialize in any particular field, but we can and must help them to develop a feeling of responsibility towards the whole of science, and its progress everywhere.

Several speakers have expressed the opinion that the formal papers were too long, and that the time for discussion was too short; but others felt that the division was about correct. Dr. John Merrill commented on the length of time required for "initiating" most of the papers; he also felt that one of the favourable aspects of this conference was the "attitude of moderation" on the part of the speakers, and the absence of the "I have all the answers" attitude.

Passing now to some of the more concrete remarks, Dr. Popper urges photoelectric observers to "provide for continuous guiding during light measurements"; the importance of this requirement was also stressed in Kron's paper.

Dr. Plaut suggests that an effort be made to observe systematically all systems down to a certain limit of apparent magnitude, in order to facilitate statistical studies. He calls attention to several bright systems which have not been properly observed, for example, H.D. 25799—a member of the Perseus association, and QS Aquilae, a visual double star discovered by Kuiper, one component of which is an eclipsing binary. A spectroscopic orbit by Hill gave a period of 2.5 days. This orbit was re-discussed by Luyten in 1936. The apparent magnitude, at maximum, is 5.6.

I should like to address to the astrometric observers the request that

they investigate again the degree to which triple and multiple systems tend towards co-planarity of their orbits. Several interesting papers on this subject have recently appeared in the *Russian Astronomical Journal*, but an independent discussion would be of great interest. Perhaps it would also be timely to study, once more, the distribution of the orbital planes in space. I hope that the photoelectric observers—especially those who have at their disposal small telescopes and whose climatic conditions are unfavourable—will devote some time to the study of changes in period of eclipsing binaries, very much in the manner of Dr. Binnendijk's work on the W Ursae Majoris stars. The systematic increases in the periods of β Lyrae and W Serpentis present a real challenge.

Dr. Popper has made a comment which I am sure we all endorse: that Dr. J. A. Pearce be requested to publish as soon as possible his vast accumulation of radial-velocity material on O- and B-type spectroscopic binaries. Many astronomers urgently need the results of his work!

In conclusion, I should like to compliment the astronomers of the Dominion Astrophysical Observatory, not only upon the enormous amount of work they have done on the orbits of many spectroscopic binaries, but especially upon their insistence upon the use of reliable wave-lengths for radial-velocity measurements. Double-star astronomy has reached the stage at which the usefulness of a spectroscopic orbit depends partly upon the consistency of the system of wave-lengths. The Victoria astronomers have given us this important tool.

VARIABLE STAR NOTES

American Association of Variable Star Observers

By MARGARET W. MAYALL, *Director*

The A.A.V.S.O. held its 45th Annual Meeting at the Springfield Museum of Natural History in Springfield, Mass., during the week-end of October 19 to 21. Our genial host, Mr. Leo Otis, Director of the Museum, and Mr. John E. Welch, Chairman of the local committee, are to be congratulated on their successful arrangements.

A large group gathered on Friday evening to hear a talk by Dr. Harlow Shapley entitled "Galaxies and What They Do to Us". The audience included a group of junior astronomers from Springfield and vicinity, and all were held entranced in typical Shapley manner throughout the evening. After the lecture, Mr. Frank Korkosz, Director of the Seymour Planetarium, gave a special demonstration of the planetarium and equipment which he designed and built.

Sessions for business and presentation of papers were held on Saturday. Abstracts of the papers, edited by R. Newton Mayall, will be published by the A.A.V.S.O. The following papers were presented: Prediction of Next Sun-spot Maximum, *J. Leith Holloway*; Sun-spot Regions on Sun, This Cycle, *Ralph N. Buckstaff*; Observation of a Star of Large Proper Motion, *George Lovi*; A Nova is Coming, *George Diedrich*; Observation of a Mysterious Object, *Herbert A. Luft*; Timing Lunar Occultations, *William G. Cleaver*; White Dwarf Stars, *John Gaustad*; Martian Seeing Seen with California Seeing, *Clinton B. Ford*; Some Interesting Light Curves, *Margaret W. Mayall*; Operation Perseid 1956, *Edward Majden*; The Boston Museum of Science Planetarium, *John Patterson*; and Remarks on Organizing Satellite Groups, *Donald Zahner*. In addition to the papers, Leith Holloway and Clinton Ford showed colour slides of California observatories, and Robert Dunn showed his colour movies of the last two A.A.V.S.O. meetings.

After the Annual Dinner, Dr. Shapley gave his traditional "High-lights of Astronomy". This year he selected them from work done in North America. Dr. Shapley said: "The artificial satellite and the breathless growth of radio astronomy have monopolized the astronomical interest of the scientific public this year. Some less spectacular contributions of high merit have been made in the older astronomical fields, and in the following list of American High-lights they compete with radio, satellites, and some new and striking hypotheses."

SOME HIGHLIGHTS OF AMERICAN ASTRONOMY IN 1955–1956

1. At the top of the list of High-lights should be the completion of the prolonged study by Milton Humason of Mount Wilson and Palomar Observatories of the radial velocities of several hundred galaxies, a work upon which we shall for long base our knowledge of the rate of expansion of the universe. The report on the work is published jointly with his colleagues, A. R. Sandage and N. U. Mayall.

2. Discovery of the anti-proton by the nuclear physicists of Berkeley, California, which gives to cosmogony a basis for strange speculations, as, for example, the

suggestion by Dr. M. Goldhaber of the Atomic Energy Commission's Brookhaven Laboratory, of an anti-matter universe quite distinct and different from our proton cosmos—a sort of mirror image of it.

3. The beginning of the world-wide organization of amateur astronomers for the visual tracking of the artificial satellite, the launching and study of which is one of the semi-astronomical projects of the International Geophysical Year.

4. The firm assurance, after three years of exploration and planning, that a large inter-institutional observatory will be located in the arid south-west, possibly in Arizona, outfitted with an 80-inch reflector and equipment for precise photometry, and with the possibility of still larger instruments in the future for the study of sun, planets, and stars. The goal is to provide instruments chiefly for the use of astronomers located at institutions with unfavourable climates or with small research facilities. This project, underwritten by the National Science Foundation, is a companion enterprise to its establishment of the 140-foot radio telescope in West Virginia.

5. A convincing astrophysical theory by W. A. Fowler and Jesse L. Greenstein of the California Institute of Technology to account for the formation of the heavy elements in stellar interiors—a contribution of high importance in our rapidly increasing knowledge concerning the evolution of stars and of the stellar universe.

6. The completion, through publication, of the University of Michigan's great programme of discovery and measurement of southern visual double stars, an enterprise of some thirty years duration, with R. A. Rossiter as the principal observer; he discovered more than 5,500 new double stars (mostly faint), which is more than any other astronomer in all past time and a record which will probably never be excelled.

7. Detection for the first time of red shifts in the radio spectrum of distant galaxies by A. E. Lilley and E. F. McLain of the Naval Research Laboratory, Washington, who found the speed of recession of a pair of galaxies (Cygnus A source) the same in the radio wave-length as in the optical measures by W. Baade and R. Minkowski—namely, about 17,000 km./sec.

8. Two special conferences of high importance at the McCormack (Virginia) and Cook (Pennsylvania) Observatories, the first to pool the continued worries of astronomers about the stellar distance scale and its revision, and the second to explore, with international participation, the future of precision instruments for measuring faint star light.

9. The dedication and putting into successful operation, under the general supervision of Dr. Bart J. Bok, of the 60-foot radio telescope at the George R. Agassiz Station of the Harvard Observatory, an instrument designed especially for research on the neutral hydrogen radiation of 21 centimetre wave-length, with the structure of the Milky Way spiral arms as one of the principal objectives.

10. The dominance in the summer and autumn sky of the planet Mars, which was in early September a mere 35 million miles distant, permitting much intense study by spectrograph, photometer, and radio, of the planet's surface at this most favourable approach in many years.

11. The announcement from the Canadian Radio Physics Laboratory at Shirley Bay, of Project Janet, a development by P. A. Forsyth and colleagues for using the ionized trains of meteors for the transmission of radio messages over long distances, at least up to 1,000 miles.

The 25th Annual Report of the Recorder showed that during the year ending September 30, 1956, 131 observers submitted 49,579 observations of variable stars in 688 reports. The following 19 countries were represented: Alaska, Argentina,

Australia, Canada, Denmark, France, Germany, Greece, Hawaii, Italy, Jamaica, Japan, Mexico, Peru, South Africa, Southern Rhodesia, Sweden, Taiwan, and the United States.

Sixteen observers made more than 1,000 observations each during the year. First and second places were retained by Reginald P. de Kock of South Africa, with 5,699, and Edward G. Oravec of New York, with 5,019 observations. The next, in order, were Fernald, Skarita, Cragg, Anderson, Taboada, Adams, Hartmann, Ford, Goldenblatt, Peltier, Aronowitz, Venter, Renner, and Darsenius.

The "Inner Sanctum" has been defined as open to observers who observe variables of 13.8 magnitude or fainter. This year twenty-three observers are eligible for membership. Work on the flare stars has proven to be of especial interest to several of our observers. Gunnar Darsenius has a record of 23 hours and 53 minutes of constant observation during the year, on eight of these intriguing objects; and he writes that he has a grand total of about 40 hours. No flares have been observed, but the observers are piling up valuable statistics.

Two annual members, Thomas A. Cragg, *California*, and Ralph A. Wright, *Massachusetts*, transferred to sustaining membership. The following 30 new annual members were elected: Jerry G. Bails, *Missouri*; M. V. Barnhill, *North Carolina*; F. J. Brinley, Jr., *Maryland*; Andre Chenier, *Ontario*; Jerome J. Domrese, *Illinois*; Max J. Edelson, *Manitoba*; James D. English, *Connecticut*; Don Estabrook, *California*; Carl E. Fowler, *Illinois*; Theodore D. Gosman, *New York*; Robert A. Hanawalt, *Ohio*; Kenneth Hicks, *Ohio*; Edward W. Hones, Jr., *South Carolina*; Norman O. Hutchings, *British Columbia*; Robert Jacobson, *Wyoming*; Anthony Jensen, *New York*; David D. Morrison, *Illinois*; O. Richard Norton, *California*; Robert C. Reinhardt, *Michigan*; Leif J. Robinson, *California*; Tom Roderick, *Ohio*; Saginaw Valley Astronomical Assn., *Michigan*; Marcos A. Severin, *Argentina*; Jerry Sherlin, *Tennessee*; Dan C. Taulman, *Texas*; John J. Thomas, *New York*; Bernard W. Trussell, *New York*; George P. Turner, *Tennessee*; Tom Waineo, *Michigan*; Curtis V. Williams, *New Jersey*.

The officers of the A.A.V.S.O. for the year 1956–57 are: President, Richard W. Hamilton; 1st V.P., Ralph N. Buckstaff; 2nd V.P., Walter P. Reeves; Director, Margaret W. Mayall; Secretary, Clinton B. Ford; Treasurer, Percy W. Witherell; and Council members William A. Miller, Virginia McK. Nail, Edward G. Oravec, John E. Welch, Harry L. Bondy, Sarah J. Hill, Dorrit Hoffleit, and Alan H. Shapley.

Observers had a chance to check on their favourite variables during a visit to the home of Mr. and Mrs. Welch. They found a number of telescopes of various sizes set up in the garden. In the house were tables laden with goodies to satisfy the inner man.

The meeting ended with a visit to the Williston Observatory of Mount Holyoke College, at the kind invitation of Dr. Alice Farnsworth. The Observatory is one of the best equipped teaching observatories in the U.S.A., and everyone was impressed with the fine facilities for elementary and advanced instruction and research projects.

A.A.V.S.O. Nova Search Report (from George Diedrich, Division Chairman): The following observations are those made in September 1956 by the person indicated with the total area-nights of searching, after the respective names: James Breckinridge, 17; Christine Culp, 12; Delorne Diedrich, 1; George Diedrich, 4; Paul Diedrich, 1; Richard H. Gemberling, 16 (all to 8.1 magnitude!); Andrew J. Keefer, Jr., 49; Beaufort S. Ragland, 7; Louis Rick, 21; Donald A. Rosenfield, 9; and Samuel L. Terry, 3. In addition Donald Rosenfield had 18 area-nights of observation in August.

Observations received during September and October: In September, 61 observers contributed 4,461 observations, and in October, 58 sent in 4,353, a total of 8,814 for the two months.

Observer	Sept. Var.	Sept. Ests.	Oct. Var.	Oct. Ests.	Observer	Sept. Var.	Sept. Ests.	Oct. Var.	Oct. Ests.
Adams, R. M.	66	135	84	207	Knowles, J. H.	21	84	...	...
Ancarani, M.	11	11	10	19	de Kock, R. P.	140	554	142	642
Anderer, J. A.	...	...	5	5	Kofoed, R.	10	31	5	7
Anderson, C. E.	201	327	210	361	Lacchini, G. B.	54	164	64	163
Arnold, J.	7	16	...	...	Lovi, G.	3	3	6	6
Beidler, H. B.	5	5	31	36	Maran, S. P.	28	42	...	...
Berg, R.	51	120	...	...	McPherson, C. A.	13	13	16	16
Breckinridge, J.	16	33	13	37	Mebius, W. G.	...	...	16	22
Brown, B.	3	5	...	...	Miller, R. W.	...	...	11	16
Buckstaff, R. N.	25	34	10	18	Montague, A. C.	9	9	29	44
Burden, J.	1	1	...	...	Morgan, F. P.	8	18	14	38
Byrd, W.	5	14	4	4	Oravec, E. G.	216	398	204	392
Carlisle, J. H.	3	6	...	...	Parker, P. O.	57	65	66	89
Carpenter, C. B.	...	...	16	17	Pearcy, R. E.	2	2	6	6
Charles, D. F.	...	...	4	11	Peltier, L. C.	20	132	28	178
Cleary, C.	5	9	...	...	Pilcher, F.	...	...	30	31
Cragg, T. A.	206	216	251	278	Price, F.	27	50	21	28
Culp, C.	7	8	23	93	Reeves, W. P.	...	...	2	3
*Darsenius, G. O.	18	143	13	122	Renner, C. J.	181	231	...	...
Diedrich, G.	2	8	7	9	Rizzo, P. V.	19	30	16	24
Erpenstein, O. M.	7	12	16	33	Roderick, T.	...	...	1	1
Fernald, C. F.	188	488	219	415	Rosebrugh, D. W.	14	106	12	90
Ford, C. B.	83	88	132	144	Royer, R.	...	...	4	4
Fowler, C. E.	...	...	3	10	Schultz, G. W.	3	3	...	...
Gemberling, R. H.	27	38	49	56	Segers, C. L.	...	...	16	43
Gleason, W. S.	3	3	...	...	Seldon, P.	8	11	...	...
Glenn, W. H.	40	72	...	...	Sharpless, A. P.	5	5	4	4
Goldenblatt, P. P.	30	30	52	52	Skaritka, P.	83	92	79	194
Goodsell, J. G.	4	4	2	2	Socha, E. M.	...	...	2	6
Halbach, E. A.	64	68	...	...	Solomon, L.	26	29	29	30
Hartmann, F.	137	154	155	162	Taboada, D.	93	104	...	...
Hein, G. J.	2	4	1	1	Taulman, D. C.	...	...	25	25
Hiett, L.	10	38	5	9	Tsai, C. H.	...	...	7	41
Houston, F. D.	3	3	...	...	Walko, K.	4	4	6	7
Howarth, M.	14	14	...	...	Walko, Mrs. K.	1	1	...	...
Hunter, T.	...	...	5	59	Waskiewicz, T.	9	9	...	...
Hutchings, N.	...	...	1	1	Weitzenhoffer, K.	...	...	7	7
Isenberg, H. D.	...	...	2	6	Wolfe, J. C.	4	8	...	...
Kelly, F. J.	10	10	11	16	Wyckoff, J.	46	115	...	...
Kimball, M.	31	31	13	13					

*Plus 20^m DO Cep; $2^h\ 23^m$ EV Lac

A.A.V.S.O. Spring Meeting, 1957: We are delighted to announce that Dr. Charles Fox has extended an invitation to the A.A.V.S.O. on behalf of the Sir George Williams College and the Montreal Centre of the Royal Astronomical Society of Canada, to hold its annual Spring Meeting in Montreal on the week-end of May 31 to June 2, 1957. A programme will be available later on for all interested persons who would like to attend.

4 Brattle Street,
Cambridge 38, Massachusetts.

November 1956

METEOR NEWS

By Peter M. Millman, *National Research Council, Ottawa*

The Relative Numbers of Bright and Faint Meteors

Regular programmes of visual meteor observations by groups rather than individuals were commenced in Canada in 1933. Since then approximately 70,000 meteors have been recorded. The great bulk of this work has been carried out by amateurs under the co-ordination and direction of the David Dunlap Observatory, Richmond Hill, and the Dominion Observatory, Ottawa. Centres of the Society have played a major part in organizing the visual observations and brief summaries of the results have appeared in the "Meteor News" columns of the Journal.

The careful estimation of the apparent visual magnitude of all meteors has always been emphasized in the Canadian programme. Some of the results of this work were presented to the American Astronomical Society at the 96th Meeting in New York, December 26–29, 1956, in a paper entitled "The Magnitude Distribution of Visual Meteors" by Peter M. Millman and Miriam S. Burland. The detailed discussion will appear shortly in the *Publications of the Dominion Observatory* but I would like here to review briefly some of the chief points in this paper.

In studying the relative numbers of meteors of various magnitudes it is customary to speak of a ratio "r", where r is defined as the ratio of the number of meteors of magnitude m to the number of magnitude $m - 1$. The greater the increase in numbers as we go to faint meteors, the higher will be the ratio r. It has been found that, for visual meteors, r is approximately constant over a normal magnitude range. Where r is constant the numbers of meteors of various magnitudes form a geometric series. If N is the number of meteors there will be a straight line relation between log N and magnitude, and the slope of this line will be log r. By application of the formulae for the summation of series it can be shown that we will get the same slope whether N represents the numbers in each individual magnitude class, or the sum of all meteors equal to and brighter than a given magnitude class—in other words the total count down to a given magnitude limit.

The relation between log N and apparent visual magnitude has been plotted in the figure for some 30,000 meteors observed on Canadian programmes during two periods. The break in observations occurred

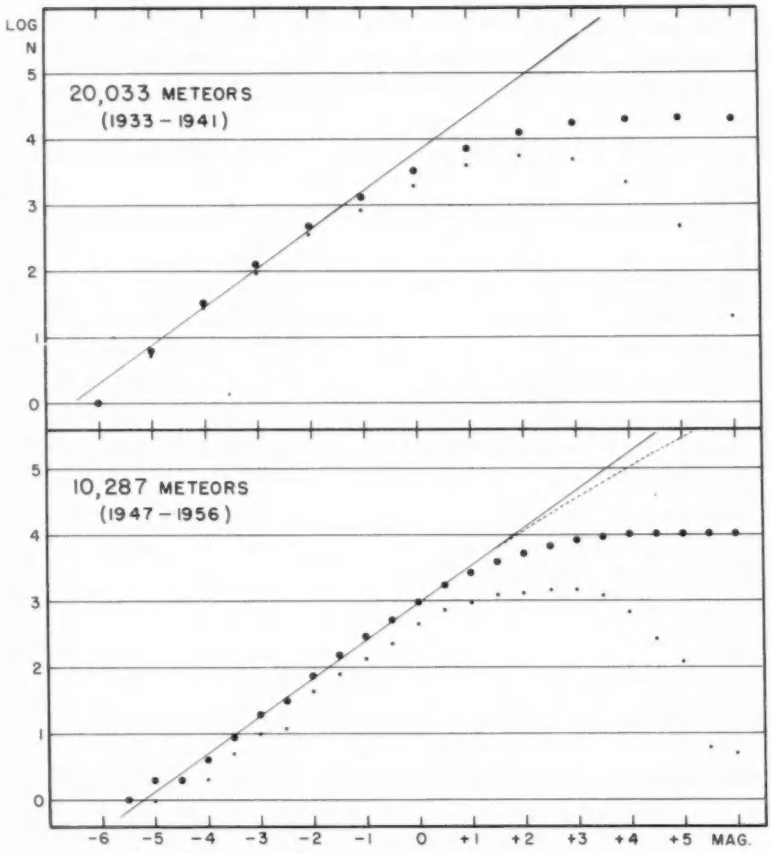

FIG. 1—The logarithm of the numbers of meteors observed plotted against their apparent visual magnitudes. The small dots refer to the actual numbers of meteors in each individual magnitude class, the large dots to the summed total of all meteors down to the indicated magnitude limit. The dotted line represents the total number of meteors on the assumption that r drops from 3.7 at magnitude 0 to 2.5 at magnitude +5. Average number of observers for 1933–1941 was 4, and for 1947–1956 was 6.

during World War II. In the earlier period magnitudes were estimated to the nearest whole number while in the later period estimates were made to the nearest half magnitude. All observations plotted in this second group were made at the Metcalfe Road Field Station near Ottawa. The slopes of the two lines are very nearly the same and the overall mean value of r is 3.7. This refers to an apparent magnitude range from -5 to $+1$ and to a mixture of shower and non-shower meteors. It is a somewhat higher value of r than has generally been assumed in the past. In observations with the naked eye it is difficult to draw conclusions about r for meteors fainter than this magnitude range as a large percentage are missed, even by a group of 6 observers watching the whole sky. Results of various programmes of telescopic meteor observation indicate that r drops to 2.5 near magnitude $+5$. On this assumption the curve for the total number of meteors will follow the dotted line in the diagram. If we allow for the faint meteors missed by the observers we find that the earth encounters roughly 200,000,000 naked eye meteors daily. These will have a minimum total mass of 10 tons but the actual mass may be considerably greater.

One of the things we would like to do on the International Geophysical Year programme is to check this magnitude relation for visual meteors over as wide a magnitude range as possible.

A New Catalogue of Meteorites

Those interested in meteroites and associated subjects will be glad to learn that a new up-to-date catalogue of the meteorites of the world has appeared recently. This is "A Classification Catalog of the Meteoritic Falls of the World" by F. C. Leonard and R. de Violini, University of California Publications in Astronomy, Vol. 2, No. 1, pp. 1–80, 1956, price $1.75 (see this Journal, vol. 50, p. 268, 1956). The catalogue includes all well authenticated meteorites known up to November 1, 1955, a total of 1,563 falls and finds. These are listed according to their classifications in the new simplified system. For cross referencing they are also listed alphabetically. This publication will be very useful, particularly for a quick check of all the known meteorites of any specified type.

NOTES FROM OBSERVATORIES

David Dunlap Observatory

Members of the Observatory staff have been saddened by the loss of our old friend, Dr. C. A. Chant, Director Emeritus, who, although retired, has been one of us since the founding of the Observatory in 1935. Our appreciation of him is expressed in an obituary elsewhere in this number of the Journal.

Observing weather was very poor during the summer, but was excellent during the autumn when both the 74-inch and the 19-inch telescopes were in operation on research programmes during most of the nights. These telescopes and the 6-inch refractor were also used more than usual for visiting groups, including the Toronto Centre of the Society and University classes. On one or two nights of particularly good seeing conditions excellent details of Mars' surface markings were observed with the 74-inch.

Good progress has been made by Dr. MacRae and his Electrical Engineering colleagues on the radio astronomy project which was referred to in earlier *Notes*. A small building has been constructed to house the receivers, and a tower and an equatorial mount have been acquired for the antenna.

Besides the regular undergraduate and graduate courses at the University of Toronto, three evening courses in astronomy are being given by members of the Observatory staff, two in the Department of University Extension and one in the Workers' Educational Association.

Dr. Peter Wellmann, who was at the Observatory during the 1955–56 session as Visiting Lecturer in Astronomy, has returned to the Hamburg Observatory.

Dr. Helen Hogg, who has returned to the Observatory after her leave of absence at the National Science Foundation in Washington, visited the Dominion Astrophysical Observatory in Victoria in August to take part in the Binary Star Conference which was arranged under the sponsorship of the Foundation. In November, Dr. Hogg visited her Alma Mater, Mount Holyoke College in South Hadley, Mass., to take part in the 75th Anniversary of the John Payton Williston Observatory.

In August, Dr. Heard presented a paper by Dr. MacRae and himself at the Binary Star Conference in Victoria and also attended the meeting of the American Astronomical Society in Berkeley, California, and visited the California Observatories.

J. F. H.

REVIEW OF PUBLICATIONS

Amateur Astronomer's Handbook by J. B. Sidgwick. Pages 580; 5½ × 8½ in. London, Faber and Faber Limited (British Book Service, Canada, Limited), 1955. Price $12.75.

This fine volume is a reference book for amateur astronomers who possess a telescope of their own. The book contains much information to guide in the use of telescopes and the methods of observing. It is not a guide to the "do it yourself" construction of telescopes, but rather a detailed description of telescopes and their accessories. The many fine illustrations and line drawings will assist in the understanding of the telescope and help in the making of the necessary adjustments to keep an instrument in good working order.

Such subjects as light grasp, magnification, size of field and the aberrations of a telescope are covered. Types of eyepieces and objectives are discussed with many illustrations to clarify the types. Telescope mountings are clearly described in detail and drives suitably explained. Optical materials and their properties, silver and aluminium films as well as other kinds of coatings for reflecting surfaces are dealt with. The use of photography, micrometers, spectroscopes and photometers in observation is detailed with many practical and useful hints.

The book contains a bibliography of many references to further information on all the subjects discussed, and a guide to the purchase of telescopes and apparatus.

Excellent printing on good quality paper characterize this fine handbook. This is the companion volume to *Observational Astronomy for Amateurs* (see this JOURNAL, vol. 50, p. 267, 1956).

G. F. L.

The Polar Aurora (International Monographs on Radio) by Carl Størmer. Pages xvii plus 403 plus 34 plates; 6½ × 9½ in. London, Oxford Press, 1955. Price 55*s*.

Prof. Carl Størmer of the Institute of Theoretical Astrophysics in Oslo has, for most of his long life, been the acknowledged authority on the subject of the aurora. He himself has been responsible for much of the observational data on auroral heights and forms, and he and Birkeland have proposed explanations for the aurora on the basis of streams of

particles. Their views are supported both by laboratory experiments and by theoretical calculations.

In *The Polar Aurora* Prof. Størmer has given a very full and clear account of both the observational data and the theoretical views, not only his own, but those of others as well. The first part of the book is easily read and it contains valuable photographs, descriptions, tables of heights, charts of frequencies of occurrence and the like. The second part, the theoretical treatment, includes a good deal of mathematics.

The Polar Aurora will undoubtedly remain for many years a standard reference book; it belongs on the desk of anyone whose work or interest is involved with the aurora.

J. F. H.

Radio Astronomy by J. L. Pawsey and R. N. Bracewell. Pages x plus 361 and 23 plates; 6½ × 9½ in. International Monographs on Radio. Oxford, Oxford University Press, 1955. Price $8.25.

Ten or fifteen years ago a book with this title could not have been written, for the science it describes is just that old. The new science now emerging has to do with the subject matter of astronomy but uses the techniques of radio and radar. Since a wide gulf of unfamiliar and specialized terms separates the old astronomy from radio, itself a new and rapidly expanding synthesis of physics and electrical engineering, someone is needed to interpret the one to the other. Drs. Pawsey and Bracewell have met this need very well.

The organization of the book follows a pattern consistent with its interpretive character. Two early chapters are devoted to techniques of observation and the theory of radio waves. This is for the astronomers and is detailed and well done. The next chapter is for the engineers explaining some aspects of the astronomy of the sun. Chapter five brings the two sciences together in a discussion of solar radio waves. Moving to the astronomy of beyond the solar system the authors discuss the structure of galaxies, and then cosmic radio waves. It is regrettable that the new scale of distances has not been adopted here.

There is a brief discussion of what radio waves generated by the heat of the moon have revealed about the moon's surface. Two chapters deal with radar techniques used in, first, extraterrestrial echoes from moon and planets and, secondly, in the observation of meteors. Finally the effect of the earth's atmosphere on radio waves is described. Here an introductory description of the ionosphere is unfortunately lacking.

In conclusion the authors speculate on the probable lines of development of this new science which is already so vigorous and productive.

The book assumes a background of physics and will make an excellent introductory text-book for radio astronomy students. It is written with clarity, enthusiasm, and the authority lent by the names of its authors, outstanding in this field.

It is one of the series "International Monographs on Radio" published by Oxford and its format is excellent with good illustrations, index, and helpful full bibliographies and references.

D. A. M. R.

NOTES AND QUERIES

On Some Relations between Comets, Meteors and Asteroids

A statistical investigation of the problem of the origin of asteroids, meteors, and comets has been undertaken by L. A. Katasev and published in the *Bulletin of the Astronomical Observatory of Stalinabad*, no. 14, p. 5, 1955. The study was based on the orbits of 144 meteors derived by Whipple and 63 orbits computed at Stalinabad. In plotting the distribution of semi-major axes of asteroids, comets and meteors, it became evident that there are no comets (with the exception of comet Encke) with a semi-major axis smaller than 2.7 A.U. and that there are not asteroids (with the exception of Hidalgo) with a semi-major axis greater than 5.7 A.U. At the same time the distribution of meteoric orbits overlaps the distribution of both the comets and asteroids.

On the other hand, a plot of eccentricities and inclinations exhibits a definite relation of meteors to comets but there are only a few asteroids with these elements falling in the region occupied by meteors.

From this investigation a conclusion has been drawn that there might exist two types of meteors—those related to comets and those related to asteroids. This, of course, brings up the question of the origin of comets and asteroids. The main difference between these two objects is in the presence of gases in comets. On the other hand there are comets known having just very thin gaseous envelopes and there are asteroids which are known to be involved in some kind of nebulosity. At any rate there is not a sharp dividing line between these objects.

As Fesenkov pointed out earlier all these objects may have had a common origin in a planet somewhere between Mars and Jupiter. At the time of disintegration some particles were thrown into parabolic orbits which became the long-period comets. Gravitational perturbations of large planets changed them gradually into comets with short periods. Assuming the original planet contained some frozen gases, these, and the lighter surface layers cemented together by ice, made up the comets while the heavier substances of the interior formed the asteroids. The influence of the sun on comets at the time of close approach would tend to disrupt them into meteors and link them with the known meteor showers. On the other hand, the debris left at the time of disintegration formed the sporadic meteors and meteorites.

Finally the question of chemical composition of meteors has to be discussed. Jacchia, Kopal and Millman have shown that the Draconid meteor shower which is connected with the comet Giacobini-Zinner is composed of matter of small density. On the other hand, Whipple thinks the Geminid shower, which as far as it is known has not been identified with any of the comets, is composed of heavier material.

As much as this view is attractive, more observations will be needed to draw a final conclusion as to the common origin of asteroids, comets and meteors.

G. A. B.

Polarized Light from the Crab Nebula

In the last decade there has been considerable publicity given to the development of large particle accelerators and it is well known that in the electron synchrotron, high energy electrons travel with velocities close to that of light in circular paths in the presence of a magnetic field. These electrons emit radiation (both radio waves and visible light). Looking tangentially to the oncoming beam of electrons in the synchrotron one can see part of this "synchrotron radiation" as a violet glow.

A similar violet glow has long been known to be emitted by the spectacular object known as the Crab Nebula, which is the remains of the supernova of 1054 A.D. reported by Chinese astronomers of the time. The Crab Nebula is also known to emit intense radio waves. Both the radio waves and the violet light lacked a convincing explanation until recently. In 1952 Shklovskii, a Russian astrophysicist, suggested that very high energy electrons exist in the Crab Nebula which spiral under the influence of the weak magnetic fields we suspect to exist there, and in so doing emit radio waves and visible light. Quantitative arguments support this suggestion.

If the visible light does originate in this way it is known that it should be polarized and that violet-sensitive photographs taken through a polaroid filter should show this. Very beautiful photographs taken by Baade with the 200-inch telescope on Palomar Mountain have recently been published (*B.A.N.*, no. 462, 1956). These taken with other rapidly increasing data entirely support the idea that the Crab Nebula emits much of its radiation by the dramatic mechanism we can observe on earth in the electron synchrotron.

Leonard Searle

Supernovae and Californium 254

Within the last few years a number of elements at the heavy end of the periodic table have been produced artificially in the laboratory. Among these is californium with an atomic weight of 254. Recently the suggestion has been made by a group of west coast scientists (W. Baade, G. R. Burbidge, F. Hoyle, E. M. Burbidge, R. F. Christie and W. A. Fowler, *P.A.S.P.*, vol. 68, p. 296, 1956) that this element may supply the energy for the outbursts of supernovae of type I. (Type I supernovae have light curves much like novae, type II supernovae fade more slowly than those of type I.)

From studies of the light curves of these supernovae it has been found that the light declines exponentially with a half life of 55 days. Also, the total energy emitted in such outbursts is of the order of 10^{49} to 10^{50} ergs, the greatest part being emitted in the first few days; the energy emitted during the exponential decay is about 10^{47} ergs.

The observed exponential decline in the light led the scientists to a consideration of the decay of a radioactive nucleus as the source of energy, for radioactive nuclei decay exponentially. Three radioactive nuclei are possible sources: an isotope of beryllium with mass 7, strontium 89 and californium 254. From considerations of the energy released from the three nuclei and of the ways in which they can be synthesized, the west coast scientists suggest that californium 254 is the most likely source. This is the only nucleus with a half life of 55 days and which decays with an energetic fission process. The scientists consider that it is probable that californium 254 would be produced in the interiors of stars that are in an advanced evolutionary stage, and are deficient in hydrogen.

R. J. N.

The Rotation Period of Venus

In a letter to *Nature* in the September 29th 1956 number, John D. Kraus of the Radio Observatory of Ohio State University has announced that an analysis of radio pulses received from Venus at a wave-length of 11 metres points to a rotation period of 22 hr. 17 min. for the planet. Kraus has found that the radio noise coming from Venus tends to reach a peak about every 13 days. Assuming that this activity is confined to one side of a rotating planet, he might have concluded that the rotation period is 13 days, but in July he observed a distinct pattern repeating itself every day for some days with a delay of about an hour and three-quarters from day to day. This has led him to conclude that the period is

really about 22¼ hours, the thirteen-day interval really representing a beat period between the rotation of Venus and the rotation of the earth.

The rotation of Venus has presented a vexing problem for many years. F. E. Ross found in 1928 that the visible markings in the cloudy surface of Venus, although suggesting a fairly short rotation period, were too evanescent for firm conclusions to be made, and Slipher and others had found in 1903 that the Doppler shift in the spectrum of the limb of Venus was so small that one could hardly believe that the rotation period was shorter than a few weeks. Ross stated, and it has been commonly repeated in text-books, that the period is probably about 30 days. The period of 22¼ hours now given by Kraus is hard to reconcile with the spectrographic observations, unless it could be that there is a large relative velocity difference between the visible surface of Venus and the parts responsible for the radio signals.

J. F. H.

Toronto Centre Bereaved

Within a period of two weeks in November 1956, the Toronto Centre was grieved by the loss of three of its officers and past officers: Dr. C. A. Chant, Honorary President, Mr. C. Frank Publow, First Vice-President, and Mr. T. H. Mason, who had been Treasurer from 1939 till 1952.

MEETINGS OF THE SOCIETY

AT WINNIPEG

January 19, 1956–The meeting was held in the Library Theatre at the University of Manitoba. The chairman was Prof. R. J. Lockhart, newly elected president of the centre.

The programme began with the unveiling of a 12½-inch mirror still in the rough stages of grinding, by Alan Wright. He and his father are in the process of building a 12½-inch reflecting telescope.

There followed an "Astronomical Quiz" in which three members of last year's class in astronomy at the University were matched against three members of this year's class. The score was a tie.

The Vice-President, Mr. G. M. Wiseman, gave a short but thorough description of the *Observer's Handbook*. Mr. Wiseman assisted in its compilation this summer. A brief question period followed.

The main topic of the evening was a talk on "Solar Energy" given by Past-President Mr. V. G. Martens. The topic of solar energy is not new since even ancient man tried to harness the sun's energy. It has become important in recent years owing to the shrinking of fossil fuels which are in such great demand today. Therefore the tendency has been to turn to the sun which strikes the earth with 4000 h.p./acre. Such attempts as the "flat head plate collector", solar engine, solar cooker and water still were discussed. The solar furnace, which is one of the greatest developments, consists of a parabolic mirror which collects the parallel rays of the sun and focuses them at a point, producing temperatures of about 3000° F. All of these devices have to do with low energy radiation, that is, the infra-red rays. Research today in this field is concerned with photosynthesis and water dissociation. Emphasis was placed on the fact that the work is still in its infancy.

After a brief question period the meeting was adjourned to the Library steps, where two 3¾-inch reflectors (Skyscopes) were set up for an observation night.

February 13, 1956–The meeting was held in the Library Theatre at the University of Manitoba. The chairman was Prof. R. J. Lockhart, who gave a brief report of the Council meeting of February 6.

Dr. R. M. Petrie, National President of the Society, was the guest speaker of the evening. He gave a description of the R.A.S.C.'s present headquarters and described the new location. It was also mentioned that the Winnipeg council had moved and passed the purchasing of this property.

Dr. Petrie's talk was entitled "Clocking the Stars", which had to do with stellar motions. Some work had been done in the past on stellar motions, but these had to do with their proper motion. About 1840–50 work in stellar motions was revolutionized by the discovery of the "Doppler Principle". When a light source sends out radiation and moves toward us the wave-lengths are changed in colour and radiation; if toward us they are shifted toward the violet end of the spectrum; if receding they are

shifted toward the red end of the spectrum. This opened a new way for measurement of stellar motions, which advanced quite rapidly until by 1954, 15,000 stars had been measured. Some of the achievements of these methods have been in the determination of the rotation of the sun on its axis, and in the discovery of spectroscopic binaries. The future in this field will lie in the determination of the motions of dimmer stars, since the brighter ones have been determined.

Dr. Petrie's lecture was illustrated with slides, which showed various famous spectroscopes including that at the D.A.O.

March 26, 1956—Once again the meeting was held in the Library Theatre at the University of Manitoba.

Mr. Evans of Victoria, the guest speaker of the evening, was introduced by the chairman. Mr. Evans' contribution to the evening was a demonstration of the new "Questar Telescope". We were shown how revolutionary Questar is in design and performance. Its versatility was revealed as Mr. Evans demonstrated its adaptation to terrestrial studies, distortionless solar work, and long-distance microscopic work.

After the meeting closed, the members were invited to examine the telescope.

April 5, 1956—The meeting was held in the Library Theatre at the University of Manitoba.

The main address was given by Dr. Peter Millman, who was introduced by the chairman. Dr. Millman's talk on the airborne solar eclipse expedition of June 30, 1954, was illustrated with slides. The reason for these airborne expeditions is due to cloudy weather destroying the ground observations. The purpose of the 1954 expedition was to photograph the spectrum of the sun, and also to see if there were any aurorae in the sky away from the sun. The chief difficulty in taking most pictures from a plane is due to vibration. Dr. Millman's talk ended with the showing of a motion picture of the eclipse.

NORMA A. TWEEDY, *Recorder*

AT MONTREAL

February 23, 1956—The meeting was held at the Physical Sciences Auditorium, McGill University, the chairman being Mr. Chas. M. Good, the President. The lecture was the "G. Horsley Townsend Memorial" lecture, in commemoration of the deep interest taken in astronomy by Mr. Townsend, and of his generous gifts to the Montreal Centre.

The speaker was Dr. Harlow Shapley, and his subject was "The Clouds of Magellan—Gateways to the Universe". The Clouds are located in the southern hemisphere of the heavens, hence are not visible in these northern latitudes. Had the early settlers to the American continent settled in the southern, instead of the northern part, our knowledge of the universe would probably be greater than it now is. Because the great observatories have been established in North America, study of the Clouds was not undertaken until later. The Clouds were reported in the accounts of Magellan's voyages as remarkable objects. They are not near the galactic plane, or connected with our galaxy.

To show the possible form of the galaxy, and the position of our solar system within it, the lecturer made extensive use of slides. Our sun is situated perhaps 28,000 light-years from the galactic centre which is obscured by dense star clouds. The elements of the structure of the galaxy were reviewed—its clusters, open and

globular, star clouds, nebulae, and the masses of obscuring matter, clouds of gas and dust all within and part of the galaxy itself. Reference was made to "Cosmography"—the study of space, time, matter and energy, and of the scale of structures from corpuscles to galaxies. Man's place in this seems to be about half-way between the star and the atom. Slides were shown of notable open and globular clusters, and their structures compared. The nebulous matter surrounding stars in the Pleiades cluster was pointed out, with the probable reasons for it. Novae were typified by the Ring Nebula in Lyra, a nebulous shell apparently the result of a great outburst of energy from the central star. There are also in the galaxy clouds of obscuring matter sufficiently dense as to cut off the light of bodies which lie beyond, and produce "Dark Nebulae" and "Coal Sack" phenomena.

In the Magellanic Clouds the same kind of stars, clusters, gaseous nebulae and novae are all found. The Clouds are considered to be independent galaxies, but different in form from our own—irregular in shape rather than spiral, and are at a distance of about 170,000 light-years. They are of much greater extent than appears; in fact, investigations using neutral hydrogen show that they nearly join.

Most important are the variables in the Clouds, of which about 1,500 are known. These include hundreds of the Cepheid type, through the study of which has been developed the Period-Luminosity-Distance law, used to measure the distances of distant objects in space. In the Greater Cloud is the remarkable "Looped Nebula", a diffuse nebula surrounding a cluster of hot, blue stars, the radiation from which causes the clouds of gas to glow. It is probably the largest diffuse nebula known and if at the distance from us as is the Orion nebula (about 600 light-years) its light would cast shadows on the earth.

Neutral hydrogen radiation is being investigated by radio astronomy. There are hundreds of such sources of radiation which are not visible to us. This mode of investigation will enable us to investigate deeper into space.

Galaxies are of several kinds—spiral, sphereoidal, and irregular. The Andromeda nebula is a typical spiral, the Magellanic Clouds are an example of the irregular type. No theory of an evolution of the form of galaxies has been proved, but rotation is believed to occur. Galaxies have a tendency to cluster, and in the Harvard survey of the northern sky, many thousands were found within a distance up to one billion light-years. The spirals form from 20 to 25 per cent. of the galaxies, although formerly the proportion was believed to be much higher. The irregular type form the smallest group.

March 8, 1956—The meeting was held at the Macdonald Physics Building, McGill University, with the President, Mr. Chas. M. Good, in the chair.

The speaker was Mr. Alan Barnes, and his subject was "The Stars". This was the third in the introductory series of lectures given by members of the Centre. Demonstrations were given of spectra, absorption lines and the characteristic lines of certain elements. The reasons for variability is stars, and for novae were discussed and slides shown to make apparent the density of stars in some sections of the heavens.

March 22, 1956—The meeting was held at Macdonald Physics Building, McGill University, with Mr. Chas. M. Good, the President, in the chair.

The speaker was Dr. P. M. Millman, of National Research Council, Ottawa. His subject was, "Solar Eclipse Expedition of June 30th, 1954". As "insurance against clouds", aerial observaions had been made. The specializations of this particular operation were:

1. Ultra-violet light during the eclipse—The chromosphere and corona.
2. Aurorae during period of totality.
3. Form of Corona.
4. Motion Pictures and Direct Photographs.

Colour photographs and films were shown of the preparations for, and during, the flights.

April 12, 1956—The meeting was held at Macdonald Physics Building, McGill University, the chairman being the President, Mr. Chas. M. Good.

The speaker was Mr. E. E. Bridgen, and his subject was "Our Stellar System and External Stellar Systems". This was the last of the Introductory series given by members of the Centre.

April 26, 1956—The meeting was held at Macdonald Physics Building, McGill University, with Mr. Chas. M. Good, the President, in the chair.

The speaker was Dr. Armand Spitz, co-ordinator of visual tracking of artificial satellites, and his subject was "Artificial Satellites". The lecture was the first official talk yet given on the Satellite Observing Programme. Not much information is available as to the satellites themselves, but an outline was given of the plans for observing, and the need for trained observers to carry through the plans developed, and some of the results desired. Some of the fields in which it is hoped to obtain information are: The upper atmosphere, radiation, and the shape of the earth with particular reference to the equatorial bulge.

May 10, 1956—The meeting was held at Macdonald Physics Building, McGill University, the chairman being the President, Mr. Chas. M. Good.

This being the closing meeting of the season, no lecture was given. The film "Hidden Treasures" was shown—an instructive and interesting nature study film, in which some astronomical matter was included. The meeting closed with a Social Hour, during which refreshments were served by the Ladies Committee.

E. E. Bridgen, *Secretary*.

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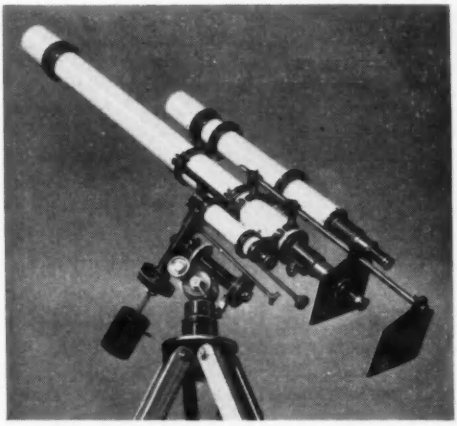

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